International Journal of Physics and Applications
Online ISSN: 2664-7583; Print ISSN: 2664-7575
Received: 02-05-2019; Accepted: 03-06-2019; Published: 09-06-2019
www.physicsjournal.in
Volume 1; Issue 2; 2019; Page No. 08-15

Marshall-olkin power Lomax distribution: A distribution for modelling life-time of products using truncated hybrid double acceptance sampling plan

Braimah Joseph Odunayo[1], Jiju Gillariose[2], Edike Nnamdi[3]
[1, 3] Department of Mathematics, Ambrose Alli University, Ekpoma, Nigeria
[2] Department of Statistics, St. Thomas College, Pala, Kerala, India

Abstract
This paper developed a truncated hybrid double acceptance sampling plan when the lifetime of a product follows a Marshall-Olkin Power Lomax distribution. In this study, the quantiles function, failure rate function, minimal sample sizes and Average Sample Number (ASN) needed for testing the quality of products, for prefixed maximum allowable percent defective, test termination ratios and acceptance numbers were obtained. The study further determines the operating characteristic values corresponding to different quality levels and the minimum ratios of the true average life to the specified life at the specified producer's risk. The study also demonstrates the results in the tables using a real life example.

Keywords: consumer's risk; producer's risk; failure rate function; quantile function; operating characteristics

1. Introduction

Acceptance sampling plan is now not only a choice or desire of the global market and industries, but a major requirement in achieving, sustaining and improving the quality of products and services so as to sustain the global business standard. Thus, it has exceedingly grown into an important tool between competitive enterprises (Subba, Naga and Kantam, 2014) [12]. An acceptance sampling plan is a specified plan in which the consumer decides whether to accept or to reject a lot of products or services shipped by the producers or consumers, based on the results of a random sample selected from that lot (Rosaiah, Kantam and Pratapa, 2007) [8]. In a typical sampling plan, one of the major challenges is how to determine the sample size that is needed to be selected from a lot under consideration. Acceptance sampling based on truncated life tests with single-item group were extensively studied by various researchers. Including, see, Baklizi (2003) [2], Baklizi and EI Masri (2004) [3], Rosaiah, Kantam and Santosh (2006) [7], Tsai and Wu (2006) [13], Srinivasa (2009) [9] and Srinivasa (2010) [10].

In real life situation, the determination of the sample size from a lot under consideration is one of the major problems in the acceptance sampling plans for a truncated life test. In this paper, a truncated hybrid double acceptance sampling plan (THDASP) is developed by considering both the producers and consumers risk. This plan was previously studied by Braimah and Osanaiye (2017) [5] as an extension of the truncated double acceptance sampling plan. Statistical literature contains some families of distributions which have been constructed based on the Marshall-Olkin extended (MOE) distributions (Marshall and Olkin, 1997) [6]. The study further determines the quantile and failure rate functions of the new distribution. The resulting new distribution gives more flexibility to model various types of data.

Marshall-Olkin extended power Lomax (MOEPL) distribution was developed by Gillariose and Tomy (2019) [4]. The cumulative distribution function of MOEPL distribution is given by

$$F(t,\alpha,\beta,\gamma,\lambda) = \frac{\alpha}{\left[1+\left(\frac{t}{\lambda}\right)^{\beta}\right]^{\gamma} - \bar{\alpha}},\ t,\alpha,\beta,\gamma,\lambda > 0 \qquad (1)$$

The corresponding probability density function is given by

$$f(t,\alpha,\beta,\gamma,\lambda) = \frac{\alpha\beta\gamma\lambda^{-\beta}t^{\beta-1}\left[1+\left(\frac{t}{\lambda}\right)^{\beta}\right]^{-\gamma-1}}{\left\{\alpha+(1-\alpha)1-\left[1+\left(\frac{t}{\lambda}\right)^{\beta}\right]^{-\gamma}\right\}^{2}},$$
$$t,\alpha,\beta,\gamma,\lambda > 0 \qquad (2)$$

The quantile function of MOEPL distribution is expressed as

$$Q(u) = \lambda\left\{\left[(1-p)^{-1}\alpha + \bar{\alpha}\right]^{\frac{1}{\gamma}} - 1\right\}^{\frac{1}{\beta}} \qquad (3)$$

Where u is generated from the Uniform (0, 1) distribution. In this paper, THDASP is performed for products life that follows MOEPL distribution. The remaining part of the paper is organized as follows: Section 2 depicts the THDASP and how to determine the minimum sample sizes of the distribution. The operating, quantiles function, hazard function and mean life ratio values in Section 3. The results are explained with some examples in Section 4. Finally, the concluding remark is given in Section 5.

2. Design of the Truncated Hybrid Double Sampling Plan

The double sampling plan entails two sample sizes (n_1 and n_2)

and also requires two acceptance numbers (c_1 and c_2). In all reviewed literature of double sampling plans including Aslam *et al*. (2009) [1], they only considered the case of c_1 = 0 and c_2 = 1.They only considered the consumer's risk, but in this study, both the producer's and consumer's risk were considered simultaneously in designing this inspection plan, which is an improvement of the work of Aslam, Jun and Ahmad (2009) [1]. This study therefore protects both the producer and consumer.

Several authors, including Braimah and Osanaiye (2017) [5], developed their own sampling plans using this point of view. The approach adopted in this study measures the quality level of a product through the ratio of its true mean lifetime to the specified length (i.e. λ/λ_o). These mean life ratios enables the producer to improve the quality of his products. From the producer's point of view, the lot acceptance probability should not be less than 1-α at the Acceptable Quality Level (AQL). Therefore, the producer emphasizes that a lot should be accepted at various levels, (λ/λ_o = 2, 4, 6, 8, 10 and 12). Conversely, from the consumer's point of view, the probability of lot rejection should be at most β.

2.1 Operating procedure for the proposed Truncated Hybrid Double Sampling Plan (THDSP)

The operational procedure of Truncated Hybrid Double Acceptance Sampling Plan (THDASP) can be illustrated as follows:

1. Select a random sample of size *n* from a lot and put them to life test at a prefixed time *t*.
2. Accept the lot if the number of defective or failed items (d) is less than or equal to the first acceptance number (c_1). Truncate or stop the test and reject the product lot as soon as the number of defective items (d) is more than the second acceptance number (c_2).
3. Whenever $c_1 < d \le c_2$, then Step 1 is repeated.

2.2 Determination of Minimum Sample Size

This approximate of minimum sample size was given by Aslam, Jun and Ahmad (2009) [1]. Using the association between Gamma and Chi-square random variable, the minimum sample size can be expressed as

$$n = \left[\frac{\chi^2_{2c+2,\,P^*}}{2P}\right] + 1 \qquad (4)$$

Where P = failure probability, c is he acceptance number and P* is the Maximum Allowable Percent Defective.

Β Is then introduced in place of P* to take care of the consumers risk while P is also replaced with $F(t,\alpha,\beta,\gamma,\lambda)$, which is the cumulative distribution function (i.e, the probability that an item in the lot will be defective during testing). Assuming the Chi-square(χ^2) random variables, equation (4) is modified as

$$n = \left[\frac{\chi^2_{v,B}}{2F(t,\alpha,\beta,\gamma,\lambda))}\right] + 1 \qquad (5)$$

Where Β takes care of the consumers risk, $F(t,\alpha,\beta,\gamma,\lambda)$ = is the failure probability and can also be taken as the producer's risk. $\chi^2_{v,\ B}$ Denotes the Β consumer's risk of a χ^2 variable with v = $2(c+1)$ degree of freedom. The approximate value of *n* can then be reduce by introducing parameter $\boldsymbol{\rho}$ (shape parameter of the failure rate). On replacing $F(t,\alpha,\beta,\gamma,\lambda)$ in (5) with $\rho F(t,\alpha,\beta,\gamma,\lambda)$, the resulting minimum sample size becomes

$$n = \left[\frac{\chi^2_{v,\beta}}{\rho F(t,\alpha,\beta,\gamma,\lambda)}\right] + 1 \qquad (6)$$

Equation (6) is the approximate of the estimated improved optimal sample size *n*.

The minimum number of items required for the THDSP in case of the MOEPL distribution are obtained at various values of the for p* = 0.25, 0.10, 0.05, 0.01 and t/λ_0 = 0.628, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712 and λ = α = 1; β = 2 are given in Table 1.

3.0 Operating Characteristics

The OC function of the sampling plan $\left(n, c, \frac{t}{\lambda_0}\right)$is the probability of accepting a lot and is given by

$$L(P) = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-1} \qquad (7)$$

The THDSP involves three parameters n, c_1 and c_2. Therefore, the operating characteristic formula for this plan is given by:

$$P = \frac{\sum_{i=0}^{c_1}\binom{n}{i}(1-p)^{n-i}}{\sum_{i=0}^{c_1}\binom{n}{i}(1-p)^{n-i} + (1-\sum_{i=0}^{c_2}\binom{n}{i}p^i(1-p)^{n-i})} \qquad (8)$$

Where *p* is the probability that an item is defective or lot fraction defective under a given product life distribution. Therefore, the operating characteristic formula for our proposed plan becomes:

$$Pa = \frac{\mathrm{Pa}}{\mathrm{Pa} + (1 - \mathrm{Pr})} \qquad (9)$$

The failure probabilities are represented by the cumulative distribution function (cdf) of the life time distributions.

3.1 Failure Rate of the Distribution

The probability density function (pdf) expresses the time till an item will completely fail. It does not directly indicate either the probability of the item continuing to work for a certain period of time or how the probability of failure is a function of the quality of the part (Srinivasa, 2011) [11]. Therefore, failure rate is defined mathematically as:

$$R(t) = Pr\,(T > t) = \int_t^\infty f(x)dx \qquad (10)$$

$= 1 - F(t)$= probability of an item meeting specification for at least till age (time t), where F (t) is the cumulative distribution function (cdf).

Therefore, a useful function used in life time analysis is the failure rate. It is defined as:

$$h(t) = \frac{f(t)}{1-F(t)} \tag{11}$$

3.2 Mean Life Ratio Value

In order to calculate the minimum required ratio values, the producer's risk is been considered. The producer's risk is the probability of rejection of the lot when $\mu \geq \mu_0$, it can be computed as follows;

$Pr(R) = \text{P(Rejecting a lot)} = 1 - P(AcceptingtheLot/\mu \geq \mu_0)$

$$= \sum_{i=c+1}^{n} \binom{n}{i} p^i \,(1-p)^{n-1} \tag{12}$$

4. Results and Examples

The results in the tables below were obtained using R software.

Table 1: Minimum Sample Sizes of MOEPL Distribution for THDASP

		$\frac{t}{\lambda_0}$							
β	c	**0.628**	**0.942**	**1.257**	**1.571**	**2.356**	**3.141**	**3.972**	**4.713**
0.25	0	2	2	2	2	2	2	2	1
	1	3	3	3	3	3	2	2	2
	2	4	4	4	4	4	3	3	3
	3	5	5	5	5	5	5	5	5
	4	6	6	6	6	5	5	5	5
	5	7	7	6	6	6	6	6	6
	6	8	8	7	7	7	7	6	5
	7	9	9	8	8	7	7	7	7
	8	10	10	8	8	7	7	8	8
	9	11	11	9	9	8	8	8	8
	10	12	12	10	10	9	9	9	9
0.10	0	2	2	2	2	2	2	2	2
	1	3	3	3	3	3	3	2	2
	2	4	4	4	4	4	4	3	3
	3	5	5	4	5	5	5	4	4
	4	6	6	6	6	5	5	5	5
	5	7	7	6	6	6	6	6	6
	6	8	8	7	7	6	6	6	6
	7	9	9	7	8	7	7	7	7
	8	10	10	8	8	7	7	7	7
	9	11	11	8	9	8	8	8	8
	10	12	12	9	10	9	9	9	9
0.05	0	2	2	2	2	2	2	1	1
	1	3	3	2	3	3	3	4	4
	2	4	4	4	4	4	4	4	3
	3	5	5	5	5	4	4	4	4
	4	6	6	6	6	5	5	5	5
	5	7	7	6	6	6	6	6	6
	6	8	8	7	7	6	6	7	7
	7	9	9	8	8	7	7	7	7
	8	10	10	8	8	8	8	8	8
	9	11	11	9	9	8	8	8	8
	10	12	12	10	10	9	9	9	9
0.01	0	2	2	2	2	2	2	1	1
	1	3	3	3	3	3	3	2	1
	2	4	4	4	4	4	4	3	2
	3	5	5	5	5	4	4	5	5
	4	6	6	6	6	5	5	5	5
	5	7	7	6	6	6	6	6	6
	6	8	8	7	7	6	6	7	7
	7	9	9	8	8	7	7	7	7
	8	10	10	8	8	8	8	8	8
	9	11	11	9	9	8	8	8	8
	10	12	12	10	10	9	9	9	9

Table 2: Average Sample Number (ASN) of MOEPL Distribution for THDASP

β	$\frac{t}{\lambda_o}$	n_1	n_2	$\frac{\lambda}{\lambda_o}=2$		$\frac{\lambda}{\lambda_o}=4$		$\frac{\lambda}{\lambda_o}=6$	
				Pa(p)	**ASN**	**Pa(p)**	**ASN**	**Pa(p)**	**ASN**
0.25	0.628	2	4	0.9033	5	0.9840	6	0.9948	6
	0.912	2	4	0.7760	5	0.9566	6	0.9853	6
	1.257	2	4	0.5903	4	0.9031	5	0.9652	6
	1.571	2	4	0.4289	3	0.8375	5	0.9382	6
	2.356	2	4	0.1586	1	0.6334	4	0.8376	5
	3.141	2	3	0.0819	1	0.5385	3	0.7865	4
	3.927	2	3	0.0258	1	0.3666	2	0.6644	3
	4.712	2	3	0.0079	1	0.2333	1	0.5384	3
0.10	0.628	2	4	0.9033	5	0.9840	6	0.9948	6
	0.912	2	4	0.7760	5	0.9566	6	0.9853	6
	1.257	2	4	0.5903	4	0.9031	5	0.9652	6
	1.571	2	4	0.4289	3	0.8375	5	0.9382	6
	2.356	2	4	0.1586	1	0.6334	4	0.8376	5
	3.141	2	4	0.0520	1	0.4291	3	0.7048	4
	3.927	2	3	0.0258	1	0.3666	2	0.6644	3
	4.712	2	3	0.0079	1	0.2333	1	0.5384	3
0.05	0.628	2	4	0.9033	5	0.9840	6	0.9948	6
	0.912	2	4	0.7760	5	0.9566	6	0.9853	6
	1.257	2	4	0.5903	4	0.9031	5	0.9652	6
	1.571	2	4	0.4289	3	0.8375	5	0.9382	6
	2.356	2	4	0.1586	1	0.6334	4	0.8376	5
	3.141	2	4	0.0520	1	0.4291	3	0.7048	4
	3.927	2	3	0.0258	1	0.3666	2	0.6644	3
	4.712	2	3	0.0079	1	0.2333	1	0.5384	3
0.01	0.628	2	5	0.8696	6	0.9767	7	0.9923	7
	0.912	2	4	0.7760	5	0.9566	6	0.9853	6
	1.257	2	4	0.5903	4	0.9031	5	0.9652	6
	1.571	2	4	0.4289	3	0.8375	5	0.9382	6
	2.356	2	4	0.1586	1	0.6334	4	0.8376	5
	3.141	2	4	0.0520	1	0.4291	3	0.7048	4
	3.927	2	3	0.0258	1	0.3666	2	0.6644	3
	4.712	2	2	0.0294	1	0.3962	2	0.6934	3

Table 3: Failure Rate of MOEPL Distribution for THDASP

B	$\frac{\lambda}{\lambda_0}$	α					
		0.01	**0.05**	**0.1**	**0.15**	**0.2**	**0.25**
0.25	2	0.4250	0.4532	0.4886	0.5240	0.5593	0.5947
	4	0.3939	0.4275	0.4696	0.5116	0.5537	0.5957
	6	0.4131	0.4503	0.4968	0.5433	0.5899	0.6364
	8	0.4353	0.4753	0.5253	0.5753	0.6253	0.6753
	10	0.4560	0.4983	0.5512	0.6041	0.6569	0.7098
	12	0.4749	0.5192	0.5745	0.6298	0.6852	0.7405
0.1	2	0.3500	0.3782	0.4136	0.4490	0.4843	0.5197
	4	0.3349	0.3652	0.4031	0.4410	0.4789	0.5168
	6	0.3410	0.3726	0.4120	0.4515	0.4909	0.5304
	8	0.3482	0.3806	0.4213	0.4619	0.5025	0.5431
	10	0.3547	0.3879	0.4294	0.4710	0.5125	0.5540
	12	0.3605	0.3943	0.4366	0.4789	0.5212	0.5635
0.05	2	0.3250	0.3532	0.3886	0.4240	0.4593	0.4947
	4	0.3170	0.3463	0.3829	0.4195	0.4561	0.4927
	6	0.3198	0.3497	0.3871	0.4244	0.4618	0.4991
	8	0.3232	0.3535	0.3914	0.4293	0.4672	0.5051
	10	0.3262	0.3568	0.3952	0.4335	0.4718	0.5101
	12	0.3288	0.3598	0.3984	0.4371	0.4758	0.5144
0.01	2	0.3050	0.3332	0.3686	0.4040	0.4393	0.4747
	4	0.3033	0.3318	0.3674	0.4030	0.4386	0.4742
	6	0.3038	0.3324	0.3682	0.4039	0.4397	0.4754

	8	0.3045	0.3331	0.3690	0.4048	0.4407	0.4765
	10	0.3050	0.3338	0.3697	0.4056	0.4416	0.4775
	12	0.3055	0.3343	0.3703	0.4063	0.4423	0.4783

Table 4: Quantile Life Function of MOEPL Distribution for THDASP

			beta =	**0.9**	**gamma =**	**0.1**		
					lambda=	**0.5**		
P*	T	$\frac{t}{\lambda_0}$	A					
			0.0100	0.0500	0.1000	0.1500	0.2000	0.2500
0.25	1	2	0.0027	0.1069	0.7086	3.2089	14.4811	70.4854
	2	4	0.0018	0.0678	0.4017	1.4923	5.1117	18.0264
	3	6	0.0014	0.0539	0.3059	1.0537	3.2400	9.9960
	4	8	0.0012	0.0465	0.2574	0.8504	2.4625	7.0404
	5	10	0.0011	0.0417	0.2274	0.7313	2.0361	5.5355
	6	12	0.0010	0.0383	0.2067	0.6520	1.7651	4.6277
	7	14	0.0010	0.0358	0.1914	0.5950	1.5766	4.0204
	8	16	0.0009	0.0338	0.1795	0.5516	1.4371	3.5847
0.10	2	4	0.0027	0.1069	0.7086	3.2089	14.4811	70.4854
	5	10	0.0018	0.0678	0.4017	1.4923	5.1117	18.0264
	2	4	0.0014	0.0539	0.3059	1.0537	3.2400	9.9960
	3	6	0.0012	0.0465	0.2574	0.8504	2.4625	7.0404
	6	12	0.0011	0.0417	0.2274	0.7313	2.0361	5.5355
	7	14	0.0010	0.0383	0.2067	0.6520	1.7651	4.6277
	3	6	0.0010	0.0358	0.1914	0.5950	1.5766	4.0204
	2	4	0.0009	0.0338	0.1795	0.5516	1.4371	3.5847
0.05	2	4	0.0027	0.1069	0.7086	3.2089	14.4811	70.4854
	5	10	0.0018	0.0678	0.4017	1.4923	5.1117	18.0264
	2	4	0.0014	0.0539	0.3059	1.0537	3.2400	9.9960
	3	6	0.0012	0.0465	0.2574	0.8504	2.4625	7.0404
	6	12	0.0011	0.0417	0.2274	0.7313	2.0361	5.5355
	7	14	0.0010	0.0383	0.2067	0.6520	1.7651	4.6277
	3	6	0.0010	0.0358	0.1914	0.5950	1.5766	4.0204
	2	4	0.0009	0.0338	0.1795	0.5516	1.4371	3.5847
0.01	2	4	0.0027	0.1069	0.7086	3.2089	14.4811	70.4854
	5	10	0.0018	0.0678	0.4017	1.4923	5.1117	18.0264
	2	4	0.0014	0.0539	0.3059	1.0537	3.2400	9.9960
	3	6	0.0012	0.0465	0.2574	0.8504	2.4625	7.0404
	6	12	0.0011	0.0417	0.2274	0.7313	2.0361	5.5355
	7	14	0.0010	0.0383	0.2067	0.6520	1.7651	4.6277
	3	6	0.0010	0.0358	0.1914	0.5950	1.5766	4.0204
	2	4	0.0009	0.0338	0.1795	0.5516	1.4371	3.5847

Table 5: Operating Characteristics Values of MOEPL Distribution for THDASP

β	$\frac{\lambda}{\lambda_0}$	n_1	n_2	$\frac{\mu}{\mu_o}$					
				2	**4**	**6**	**8**	**10**	**12**
0.25	0.628	2	4	0.90331	0.98396	0.99481	0.99772	0.99880	0.99930
	0.912	2	4	0.77596	0.95661	0.98532	0.99340	0.99649	0.99792
	1.257	2	4	0.59034	0.90313	0.96521	0.98393	0.99132	0.99480
	1.571	2	4	0.42886	0.83754	0.93821	0.97068	0.98393	0.99028
	2.356	2	4	0.15863	0.63342	0.83762	0.91724	0.95278	0.97069
	3.141	2	3	0.08188	0.53845	0.78646	0.88957	0.93663	0.96058
	3.927	2	3	0.02581	0.36655	0.66439	0.81451	0.88952	0.92966
	4.712	2	3	0.00787	0.23325	0.53837	0.72683	0.83076	0.88954
0.10	0.628	2	4	0.90331	0.98396	0.99481	0.99772	0.99880	0.99930
	0.912	2	4	0.77596	0.95661	0.98532	0.99340	0.99649	0.99792
	1.257	2	4	0.59034	0.90313	0.96521	0.98393	0.99132	0.99480
	1.571	2	4	0.42886	0.83754	0.93821	0.97068	0.98393	0.99028
	2.356	2	4	0.15863	0.63342	0.83762	0.91724	0.95278	0.97069
	3.141	2	4	0.05196	0.42909	0.70483	0.83766	0.90324	0.93826
	3.927	2	3	0.02581	0.36655	0.66439	0.81451	0.88952	0.92966
	4.712	2	3	0.00787	0.23325	0.53837	0.72683	0.83076	0.88954

0.05	0.628	2	4	0.90331	0.98396	0.99481	0.99772	0.99880	0.99930
	0.912	2	4	0.77596	0.95661	0.98532	0.99340	0.99649	0.99792
	1.257	2	4	0.59034	0.90313	0.96521	0.98393	0.99132	0.99480
	1.571	2	4	0.42886	0.83754	0.93821	0.97068	0.98393	0.99028
	2.356	2	4	0.15863	0.63342	0.83762	0.91724	0.95278	0.97069
	3.141	2	4	0.05196	0.42909	0.70483	0.83766	0.90324	0.93826
	3.927	2	3	0.02581	0.36655	0.66439	0.81451	0.88952	0.92966
	4.712	2	3	0.00787	0.23325	0.53837	0.72683	0.83076	0.88954
0.01	0.628	2	5	0.86960	0.97673	0.99229	0.99656	0.99818	0.99893
	0.912	2	4	0.77596	0.95661	0.98532	0.99340	0.99649	0.99792
	1.257	2	4	0.59034	0.90313	0.96521	0.98393	0.99132	0.99480
	1.571	2	4	0.42886	0.83754	0.93821	0.97068	0.98393	0.99028
	2.356	2	4	0.15863	0.63342	0.83762	0.91724	0.95278	0.97069
	3.141	2	4	0.05196	0.42909	0.70483	0.83766	0.90324	0.93826
	3.927	2	3	0.02581	0.36655	0.66439	0.81451	0.88952	0.92966
	4.712	2	2	0.02942	0.39624	0.69337	0.83564	0.90425	0.94003

Table 6: Minimum Ratio Values for MOEPL Distribution

		$\frac{t}{\lambda_0}$							
β	c	**0.628**	**0.942**	**1.257**	**1.571**	**2.356**	**3.141**	**3.972**	**4.713**
0.25	0	26.518	27.480	26.518	27.563	24.582	29.533	27.563	31.586
	1	7.587	7.587	7.525	7.169	7.587	7.342	7.587	7.525
	2	4.690	4.715	4.690	4.527	4.417	4.573	4.460	5.084
	3	3.654	3.698	3.654	3.554	3.513	3.554	3.625	4.155
	4	3.115	3.105	3.053	3.053	3.042	3.042	3.012	3.433
	5	2.799	2.756	2.756	2.748	2.660	2.732	2.636	3.022
	6	2.548	2.555	2.499	2.478	2.485	2.425	2.394	2.740
	7	2.394	2.381	2.362	2.338	2.350	2.291	2.314	2.644
	8	2.257	2.251	2.218	2.229	2.197	2.186	2.171	2.478
	9	2.150	2.150	2.140	2.110	2.076	2.100	2.058	2.356
	10	2.081	2.067	2.048	2.044	2.026	1.986	1.969	2.251
0.10	0	44.170	43.122	44.170	43.328	44.170	41.357	41.357	47.148
	1	10.593	10.718	10.471	10.593	10.471	9.901	9.588	10.977
	2	6.365	6.321	6.150	6.234	5.949	6.028	5.760	10.977
	3	4.789	4.715	4.666	4.596	4.619	4.439	4.417	6.596
	4	3.956	3.922	3.905	3.872	3.823	3.807	3.729	5.028
	5	3.459	3.459	3.394	3.369	3.332	3.308	3.320	4.272
	6	3.126	3.137	3.105	3.032	3.022	2.982	2.934	3.791
	7	2.897	2.888	2.851	2.790	2.799	2.748	2.756	3.357
	8	2.699	2.699	2.675	2.660	2.629	2.577	2.548	3.159
	9	2.569	2.555	2.527	2.520	2.499	2.438	2.452	2.916
	10	2.445	2.445	2.419	2.375	2.399	2.384	2.224	2.807
0.05	0	56.529	56.883	55.835	55.157	54.171	53.220	55.157	63.211
	1	13.021	13.021	12.658	12.837	12.484	12.484	12.658	14.472
	2	7.463	7.323	7.283	7.225	7.225	7.169	7.057	8.058
	3	5.516	5.549	5.417	5.322	5.353	5.322	5.200	5.910
	4	26.518	27.480	26.518	27.563	24.582	29.533	27.563	31.586
	5	7.587	7.587	7.525	7.169	7.587	7.342	7.587	7.525
	6	4.690	4.715	4.690	4.527	4.417	4.573	4.460	5.084
	7	3.654	3.698	3.654	3.554	3.513	3.554	3.625	4.155
	8	3.115	3.105	3.053	3.053	3.042	3.042	3.012	3.433
	9	2.799	2.756	2.756	2.748	2.660	2.732	2.636	3.022
	10	2.548	2.555	2.499	2.478	2.485	2.425	2.394	2.740
0.01	0	2.394	2.381	2.362	2.338	2.350	2.291	2.314	2.644
	1	2.257	2.251	2.218	2.229	2.197	2.186	2.171	2.478
	2	2.150	2.150	2.140	2.110	2.076	2.100	2.058	2.356
	3	2.081	2.067	2.048	2.044	2.026	1.986	1.969	2.251
	4	44.170	43.122	44.170	43.328	44.170	41.357	41.357	47.148
	5	10.593	10.718	10.471	10.593	10.471	9.901	9.588	10.977
	6	6.365	6.321	6.150	6.234	5.949	6.028	5.760	10.977
	7	4.789	4.715	4.666	4.596	4.619	4.439	4.417	6.596

	8	3.956	3.922	3.905	3.872	3.823	3.807	3.729	5.028
	9	3.459	3.459	3.394	3.369	3.332	3.308	3.320	4.272
	10	3.126	3.137	3.105	3.032	3.022	2.982	2.934	3.791

4.1 Example

The demonstration of tabulated results in this study is carried out using Lead Acid Battery failure life data as extracted from the record of Uyi Technical Batteries, Jattu Road, Auchi, Edo State, Nigeria (Braimah and Osanaiye, 2019) [5].

Table 7: Battery Current Drain of Lead Acid Battery

Manufacturer	Discharging Ampere	Time from 100% Charged to 0% Discharged (Minutes)				
Universal Power Group	(12V) 24Ah	40	83	18	38	19
Canbat Batteries	(12V) 18Ah	60	56	35	75	38
Tenergy	(12V) 10Ah	42	42	53	113	25
National Battery	(12V) 24Ah	28	28	70	70	34
Avon Battery	(12V) 24Ah	21	19	24	119	23
Sigmas Battery Tek	(12V) 18Ah	14	62	20	33	12
Energy	(12V) 19Ah	19	43	27	38	43

The sample mean life of the batteries is approximately 42. (i.e, λ_o = 42). Assuming the testing time is 180 minutes, i.e the time for initial voltage dip will be 180 minutes. The experimental time ratio to the mean life is $\frac{t}{\lambda_0} = \frac{180}{42} = 4.2857$.

Suppose the Industry want to adopt THDASP as a Sampling Plan in order to determine whether the life of Lead-Acid Batteries are above the specified average life of 42 with Maximum Allowable Percent Defective of P* = 0.10 and the life test would end at 180 minutes, which should have led to an approximate experimental ratio of $\frac{t}{\lambda_0} = 4.2857$.

Assuming that the lifetime of products follows a MOEPL Distribution, given an acceptance number $c_1 = 2$ and $c_2 = 3$, the designed parameters of the Sampling Plan are $\left(n, c_1, c_2, \frac{t}{\lambda_0}\right) = 7, 2, 3$ and 4.2857 for P^*= 0.90. That is from table 1, the experimenter needs to select a sample of 3 and 4 products respectively and put them to test, the lot is rejected if more than 2 and 3 failures occur during the first and second testing during 180 minutes, otherwise accept it.

For P^* = 0.10, under MOEPL distribution with $\frac{\lambda}{\lambda_0} = 2$, from table 5, the probability of acceptance for the plan is shown in Table 8 and Figure 1 (operating characteristics curve) below:

Table 8: Probability of Acceptance values for Lead-Acid Experiment

Product Quality $\left(\frac{\lambda}{\lambda_0}\right)$	**2**	**4**	**6**	**8**	**10**	**12**
Probability of Acceptance (P)	0.02581	0.36655	0.66439	0.81451	0.88952	0.92966

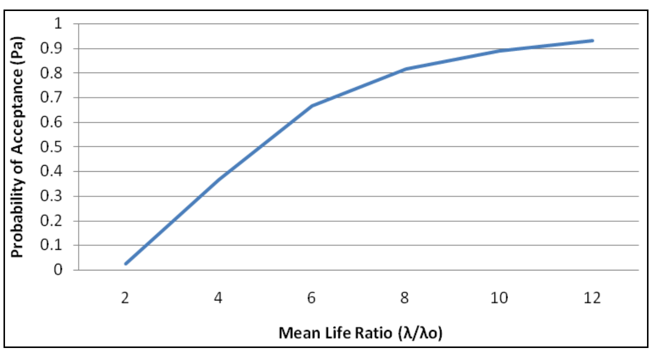

Fig 1: Operating Characteristics (OC)

It is observed from Table 8 and Figure 1 above that for lots of products to be accepted at a higher probability, the producer needs to increase the mean life of his product also.

5. Concluding Remarks

A hybrid double acceptance sampling plan for a truncated life test based on MOEPL distribution was proposed. The optimal number of items to be selected from the lot and be put to test, failure rate function, quantile function, operating characteristics and mean life ratio values were determined for the distribution, with other designed parameter (shape parameters) of the distribution prefixed.

Based on the findings of this study, the optimal sample number of items needed to be selected from the lot decreases as the experimenting time increase. Furthermore, the operating characteristic values increases as the mean life ratio increases, which shows that items with increased mean life will be accepted at a higher probability compared with the ones with lower mean life ratio. Afterward, the study expanded with a real data set using Lead-Acid Battery for illustration.

Conclusively, the plan may provide quality control experimenter in manufacturing industries a more economical mechanism for reducing testing time, cost, energy and labour. Hence, this plan may serve as an alternative sampling plan to other existing sampling plans.

References

1. Aslam MC, Jun H, Ahmad M. Double acceptance sampling plans based on truncated life tests in the Weibull model, J. Stat. Theor. Appl. 2009; 8(2), 191-206
2. Baklizi A. Acceptance sampling based on truncated life tests in the Pareto distribution of the second kind. Advances and Applications in Statistics. 2003; 3(1):33-48.
3. Baklizi A, EI Masri AEK. Acceptance sampling based on truncated life tests in the Birnbaum-Saunders model. Risk Analysis. 2004; 24(6):1453-1457.
4. Gillariose J, Tomy L. The Marshall-Olkin Extended Power Lomax Distribution with Applications, (preprint), 2019.
5. Braimh JO, Osanaiye PA. Truncated Hybrid Double

Acceptance Sampling Plan (THDASP) for Weibull Product Life Distribution, American Journal of Management Science and Engineering. 2017; 2(5):80-88.
6. Marshall AW, Olkin I. A new method for adding a parameter to a family of distributions with application to the exponential and Weibull families. Biometrica, 1997; 84:641-652.
7. Rosaiah K, Kantam RRL, Santosh KH. Reliability of test plans for exponentiated log-logistic distribution, Economic Quality Control. 2006; 21(2):165-175.
8. Rosaiah K, Kantam RRL, Pratapa RJ. Economic reliability test plan with Inverse Rayleigh Variate, Pakistan Journal of Statistics. 2007; 24(1):57-65.
9. Srinivasa RG. A group acceptance sampling plans for lifetimes following a generalized exponential distribution, Economic Quality Control. 2009; 24(1):75-85.
10. Srinivasa RG. A group acceptance sampling plans for truncated life tests for Marshall-Olkin extended Lomax distribution, Electronic Journal of Applied Statistical Analysis. 2010; 3(1):18-27.
11. Srinivasa RG. A Hybrid Group Acceptance Sampling Plans for Lifetimes Based on Generalized Exponential Distribution, Journal of Applied Sciences. 2011; 11(12):2232-2237.
12. Subba RR, Naga DA, Kantam RRL. Hybrid Group Acceptance Sampling Plan Based on Size Biased Lomax Model, Mathematics and Statistics. 2014; 2(3):137-141.
13. Tsai TR, Wu SJ. Acceptance sampling based on truncated life tests for generalized Rayleigh distribution, Journal of Applied Statistics. 2006; 33(6):595-600.