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Reducing the short pulses stretching caused by lenses

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Abstract
A short optical or x-ray pulse focused by a system of lenses is stretched in focus due to several reasons. In this paper the material effect is discussed, which occurs due to retardation of the pulse when propagating inside a lens. Two methods are proposed to minimize these effects. The first one is to insert the compensation element in front of the lens. The second method is to use an unusual "up-down" lens combination. These methods keep the focusing distance reasonably short.

Keywords: short laser pulses, Short x-ray pulses, lenses for short pulses, double lens system, FEL

Introduction

A number of applications in science require the use of short or ultra-short pulses, not only in the optical region but also e.g. in the X-ray region. Such pulses are generated by lasers or free-electron lasers (FEL). The optical systems should be designed in such a way that they keep the focused pulses as short as possible. Focusing optics usually has significant influence on the spatial and temporal characteristics of the pulse in the focal region. The best known focusing system is a lens. This has been known for a long time and its properties are described in detail for example in [1]. Next to this, X-ray lenses are known since 1996 [2] [3]. Today, a huge number of papers exists that dealing with physics and applications of X-ray lenses (see e.g. [4] and the citations therein). Lenses may be destroyed by high power laser systems.

Focused ultra-short pulse is a superposition of many spectral components in spacetime. Each component creates focus in a slightly different place, which contributes to pulse stretching. This problem has been solved (see e.g. [5, 6, 7, 8] and thus will not be treated in this paper.

The propagation of the electromagnetic radiation through lenses may be studied by ray tracing method, or more precisely by wave optical method [9]. According to [10] the results obtained by ray tracing and wave optical analysis are in good agreement. For this reason the considerations in this paper are based on geometrical models only.

In this short paper two ideas are proposed to suppress pulse stretching due to the geometrical and material effects. An emphasis is laid on the focused pulse length rather than on the properties of the focus. The calculations are performed for thin lenses.

The pulse deformation by a lens and its minimization

The distortion of short laser pulses in optical lenses was studied in [11] and [12]. There are basically two reasons for pulse stretching in a focus (besides of chromaticity). The first one is due to the path length difference among various rays between the lens and the focus. For example, the marginal ray impinging at point A has a longer way to propagate to focus F Than the ray impinging to B (see Fig. 1). This will be called the geometrical effect. The second one is due to retardation of rays inside the lens. The pulse inside the lens propagates by group velocity v_g [13] which is lower than the phase velocity v_p. (The corresponding refraction indices are n_g and n_p.) Various rays have various paths inside the lens which give rise to pulse stretching in the focus. This will be called the material effect. For long monochromatic wave the geometrical effect fully compensates the material effect. The pulse stretching is due to the difference between n_g and n_p. This is a well-known effect [14]. Group and phase refractive indices are related by the slope of the dispersion curve [15].

$$n_g - n_p = -\lambda dn_p/d\lambda, \quad (1)$$

Where λ is vacuum wavelength.

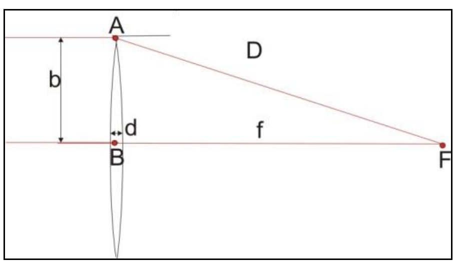

Fig 1: The optical lens.

The refractive index n_p may be calculated from Sellmeier equation [14]. If the thickness of the lens is d, then $(n_g - n_p)\, d$ represents the additional optical path length which is not compensated by the geometrical effect. For example, if λ= 0.6 μm, d = 5mm, and the lens material is BK7 glass, the refraction indices are n_p = 1. 516, $(n_g - n_p)$ = 0.024 and the additional optical path lengths is 0.12 mm. The time delay between the axial and marginal ray is about 400 fs. For shorter λ the delay is higher. Similar results are presented in [11].

The part of the material effect which is not compensated by the geometrical effect may be partially eliminated by inserting a compensation element in front of the lens from the same material as the lens (Fig. 2). This element should be designed such that the sum of its thickness for certain beam multiplied by n_g and the beam path length inside the lens multiplied by (n_g - n_p) is approximately constant for all beams. For the above example the thickness of the element for marginal rays should be 0.12/1.516 mm. For axial ray the thickness should be zero. The material of the compensation element may be different but its "optical thickness" should be (n_g - n_p) d. In reality, such compensation element may diffract, which would complicate the compensation effect.

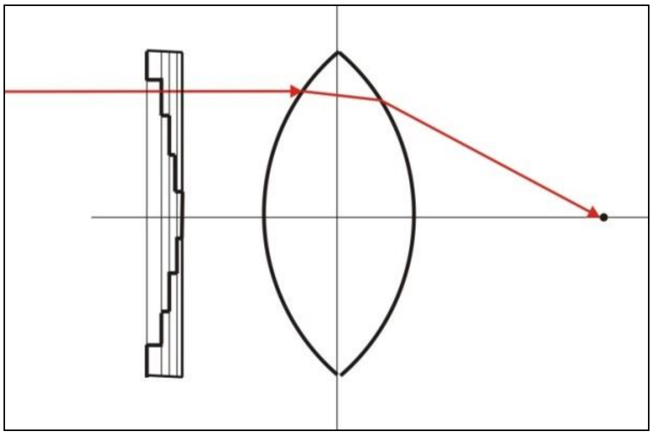

Fig 2: The compensation element in front of the lens suppresses the material effect. It should be designed such that the sum of its "optical thickness" for certain beam and the beam path length inside the lens multiplied by (n_g - n_p) is approximately constant for all beams.

In the X-ray region the material and geometrical effects are much smaller than in the optical region. This is due to the fact that the refraction index is very close to 1. In the optical region the ray at the lens axis (axial ray) has minimal path length (BF) but it has maximal retardation in the lens. This means that the geometrical effect partially suppresses the pulse stretching caused by the material effect. In the X-ray region the lens has a concave shape and the optical path length of the beam at the lens axis is minimal (taking into account the group velocity). Here both effects contribute to pulse stretching. (It is evident that to minimize the pulse stretching a thin lens with long focusing distance should be used.)

In X-ray region the refraction index n_p can be calculated as [16]

$$n_p = 1 - \delta, \tag{2}$$

Where

$$\delta = r_e N \lambda^2 / 2\pi. \tag{3}$$

By substitution of (3) into (1) we find:

$$n_g = 1 + \delta \tag{4}$$

Or

$$v_g v_p = c^2 , \quad n_g n_p = 1. \tag{5}$$

As δ is of the order of 10^{-5}, then n_g - n_p = 2δ, and it could be taken approximately as 10^{-4}.

X-ray compound refractive lens (CRL) is mostly composed of several tens of single refractive lenses. The overall thickness of CRL for marginal rays may be from several mm to several cm. By multiplying this value by 10^{-4} (=n_g - n_p), we get the thickness of the compensation element for the central ray 0.5 ÷ 5 μm. Without the compensation element the pulse stretching is several fs. These values may be acceptable for fs pulses, but are problematic for shorter pulses.

Another (and probably better) way of compensation consists in using a rather unusual arrangement of lenses (Fig. 3a).

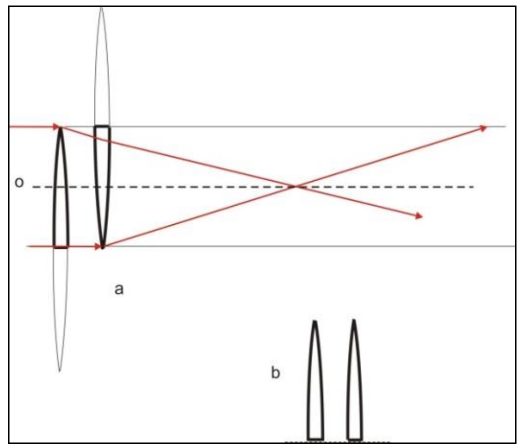

Fig 3: A scheme showing two cylindrical (2D) lenses. The "up-down" arrangement (a) suppresses the material effect (see text). as compared with the usual arrangement (b).

As is shown in appendix, such a lens combination (up-down arrangement) is a focusing system. The variation of total path Length in both lenses for every beam is here minimal. In other words, both lenses partially compensate each other. It is obvious that this arrangement suppresses the material effect which may be strong in the usual arrangement shown in Fig. 3b. The pulse stretching is shortened approximately four times for thick lenses as compared with the arrangement shown in Fig. 3b. For thin lenses the stretching should be even shorter. (The mutual distance of the lenses is supposed to be smaller than the focusing distance and should be as small as is technically possible.) From appendix (eqs. (7), (8)) it also follows, that the mutual distance of lenses should be small, because in this case the wavelength dependence of the focus position is minimal (dx(λ) ≈ df(λ)/2). Fig. 3 is only a schematic 2D picture corresponding to cylindrical lenses. To create a real 3D lens system, it is necessary to rotate the lens system (Fig.3) about the axis *o* passing through the focus by 180^0. It leads to very unusual shape as shown schematically in Fig. 4. (Here the y coordinate of the focus is R/2, which holds for small s).

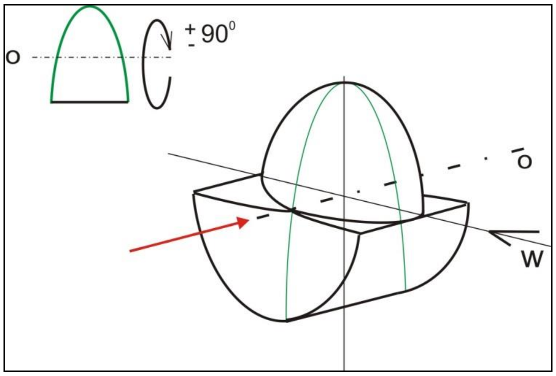

Fig 4: An example of 3D lens originating by rotation of the lens profile around the axis o by 180^0. Fig. 3 is the view from direction **w** before rotation.

This is a complicated shape. Fortunately, the lens system shown in Fig. 4 may be modified as shown in Fig. 5. The modified system is symmetrical with respect to the axis o and thus it represents the 3D system.

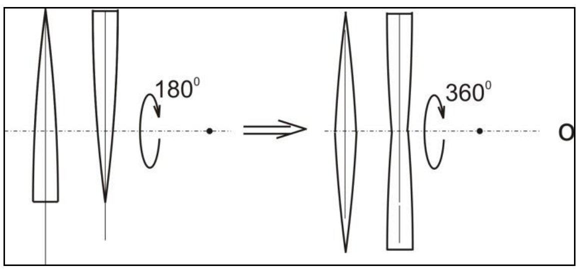

Fig 5: A modification of the lens system shown in Fig. 3. It represents a 3D lens system which is symmetrical with respect to the axis o. The system is focusing except of rays impinging close to axis o. These rays, when leaving the system, are divergent and may be shielded.

Principally, the “up-down” modification may be also applied to an X-ray lens. The only difference is that the X-ray lens has concave shape. For example, the very first compound refractive X-ray lens [2] consists in holes drilled into rod. The axes of the holes were parallel and were located in one plane. The space between two holes acts as a lens. This could be modified as shown in Fig. 6 to create an “up-down” arrangement.

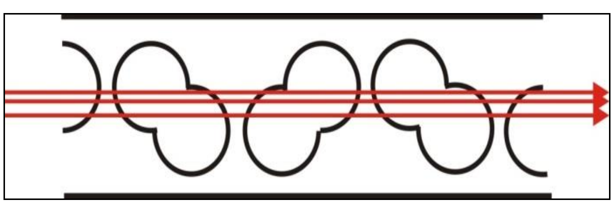

Fig 6: The modification of the very first X-ray lens to create an „up-down“ arrangement.

Conclusion

The focusing of short pulses by classical lens system leads to pulse stretching in the focus. In this paper a modification of the lens system is proposed (up-down arrangement) which minimizes this stretching. The stretching is shortened approximately four times or even more for thin lenses.

Appendix

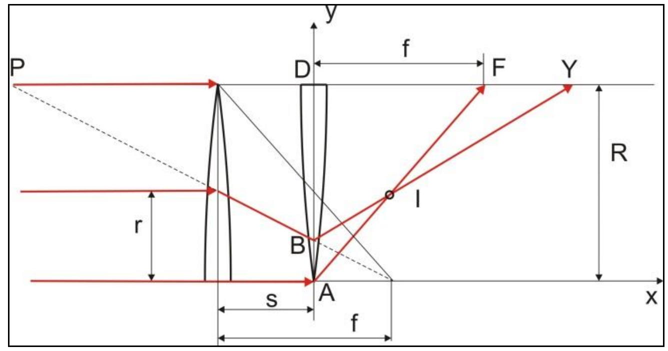

Fig 7: The “up-down” arrangement of two identical lenses.

It is supposed that both lenses are identical. The focusing distances are f. The parallel beam is impinging on the first of them. The task is to find the coordinates od the intersection point I for various r. In thin lens approximation it holds:

$$1/PD + 1/DY = 1/f. \quad (6)$$

The coordinates of points A, B, D, F, P, Y are following:

A(0, 0), B(0, r(f-s)/f), D(0, R), F(0, f),

P(-Rf/r + (f-s), R), Y([Rf – r(f-s)]f/[Rf – r(2f – s)], R).

The coordinates of the point I may be calculated as the intersection of lines AF and BY. The coordinates of the point I are following:

$$x = f(f-s) / (2f – s),$$
$$y = R(f-s) / (2f – s). \quad (7)$$

It is seen that these coordinates are independent of r and thus the point I is a focus.
The focusing distance f (□) is a function of wavelength. The variation of x with the wavelength may be seen from the following equation:

$$dx/df = 1 – 2f(f-s) / (2f-s)^2. \quad (8)$$

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