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Superluminal, contradiction and unification between quantum theory and special relativity

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Abstract
First, some superluminal phenomena are introduced. Next, based on the complete special relativity which includes the generalized Lorentz transformation (GLT) and superluminal in the spacelike interval, we search contradiction between the uncertainty principle in quantum mechanics and strict classification of the timelike-spacelike intervals in special relativity. Third, we propose new interpretation of the duality in the double-slit experiment that is the particle-wave duality corresponds to the timelike and spacelike intervals, respectively. Such the duality is mutually exclusive and complementary. Fourth, we derive the new LT and GLT for very small space and time, and research possible unification of quantum theory and complete special relativity.

Keywords: Quantum theory, special relativity, superluminal, particle-wave duality

1. Introduction
According to a standard interpretation of the special relativity, the superluminal is exactly excluded. But, much superluminal phenomena are continuously discovered.
In 1981, Pearson, *et al.*, discovered that maps of the radio structure of quasar 3C273 shown directly that it expanded with an apparent velocity 9.6 times the speed of light [1]. Other radio sources possess also superluminal expansion. These phenomena may be explained by the relativistic beaming model [2, 3], etc. Since 1995 many astronomers in different countries discovered some superluminal motions [4].
Scharnhorst researched the propagation of light in the vacuum between plates, and their impact phrased in the simplest terms consists in causing a change in the velocity of light [5]. Barton used Euler-Heisenberg interaction, and derived the Scharnhorst effect and faster than velocity of light between parallel mirrors [6]. In this case,

$$c' = c/n \approx (1-\Delta n)c > c, \tag{1}$$

in which $\Delta n = -(11\pi^2/8100)(\alpha^2/m^4L^4) \approx -1.55\times 10^{-48}(cm/L)^4$, here L is distance between plates. But, the effects are too small by many orders of magnitude to be measured. Then Steinberg, *et al.*, measured the superluminal group velocity in the single-photon tunneling time [7], and observed highly superluminal propagation in a medium with a gain doublet [8]. Aharonov, *et al.*, pointed out, unstable systems as media with inverted atomic population have been shown to allow the propagation of analytic wave packets with group velocity faster than that of light, without violating causality [9]. In 2000, Mugnai, *et al.*, observed superluminal behaviors in the propagation of localized microwaves over distances of tens of wavelengths, and all of the components in the spectral extension have the same propagation velocity $v = c/\cos\theta$ [10]. Wang, *et al.*, measured a very large superluminal group velocity that exceeds about 310 times faster than the speed of light in a vacuum, but these light pulses do not violate causality [11]. Marangos reviewed various superluminal effects [12]. Therefore, we must face squarely superluminal [13]. Recently, Wang, *et al.*, studied limiting superluminal neutrino velocity and Lorentz invariance violation by neutrino emission from the blazar TXS 0506+056 [14]. Gusikhin, *et al.*, discussed superluminal electromagnetic two-dimensional plasma waves [15].
Based on the advances in high-speed imaging techniques, Morimoto, *et al.*, demonstrated the four-dimensional light-in-flight imaging on the observation of a superluminal motion captured by a new time-gated megapixel single-photon avalanche diode camera [16].

Tasgin investigated the negative superluminal velocity and distinguish between two kinds of superluminosity, where with the violation of Kramers-Kronig relations in causal optical systems [17]. Ramón, *et al*., studied the relation between faster-than-light signaling and the causal factorization of the dynamics for multiple detector-field interactions, and predicted superluminal propagation of the field's initial conditions, and discussed the validity of measurements in quantum field theory with nonrelativistic particle detectors [18]. Tong, *et al*., studied semiclassical communication in positivity-violating k-essence scalar field theories, and derived a novel bit rate bound on superluminal communication within a conceptual model, whose result implies the possibility that, even if these positivity-violating k-essence theories may not possess a maximal information propagation speed, there is nevertheless an upper bound on the rate of information transfer [19].
Li, *et al*., discussed ultrabright electron bunch injection in a plasma Wakefield driven by a superluminal flying focus electron beam [20]. Chen, *et al*., simulated superluminal propagation of Dirac particles using trapped ions [21]. Apostolakis, *et al*., studied beyond the ordinary acoustoelectric effect as superluminal phenomena in the acoustic realm and phonon-mediated Bloch gain [22]. Vilasini, *et al*., proposed impossibility of superluminal signaling in Minkowski spacetime does not rule out causal loops [23].
Moreover, some superluminal phenomena exist also in quantum and microscopic domain, such as quantum entanglement, where an electron appears simultaneously at two separated sites. Superluminal may also exist in the tunneling effect through the barrier, etc. In this paper, based on the complete special relativity, we search contradiction between the uncertainty principle and strict classification of the timelike-spacelike intervals, and propose new interpretation of the duality and their possible unification.

2. Complete Special Relativity

Based on the classification of the timelike and spacelike intervals, we may derive two symmetrical topological structures separated by the light-cone [24, 13]:
A. The timelike interval $s^2 = r_{mn}^2 - c^2 t_{mn}^2<0$; the speed defined as $|v| = |r_{mn}/t_{mn}|<c$ (subluminal); the time dilation must be at the same space position, x=0, from which $s^2 = x^2 - c^2t^2 = -c^2T^2$, and $t = T/\sqrt{1-(v/c)^2}$.
B. The spacelike interval $s^2 = r_{mn}^2 - c^2 t_{mn}^2>0$; the speed defined as $|v| = |r_{mn}/t_{mn}|>c$ (superluminal); the length contraction must involve only one time instant, t=0, from which $s^2 = {l_0}^2 - c^2t^2 = R^2$, and $R = l_0\sqrt{1-(c/\bar{v})^2}$.
Based on the basic principles of the special relativity, according to the constancy of the velocity of light in the vacuum principle, $s^2 = r_{mn}^2 - c^2 t_{mn}^2$ is invariance. In a time-space ($x_0 - x_1$) plane, a universal transformation of the special relativity is:

$$x_1{}' = x_1 ch\phi - x_0 sh\phi, \tag{2}$$

$$x_0{}' = x_0 ch\phi - x_1 sh\phi. \tag{3}$$

When one adds an independent hypothesis: At the same space position (for example, the origin of K' frame, or for rest point, etc.), namely the premise x'=0 always exists. So from (2) $th\phi = x_1/x_0 = v/c$ is obtained, then we derive the Lorentz transformation (LT):

$$x_1{}' = \gamma(x_1 - vt), t' = \gamma(t - vx_1/c^2), \tag{4}$$

where $\gamma = 1/\sqrt{1-(v/c)^2}$. Various approaches of introducing the Lorentz transformation, without exception and unanimously, applied this similar hypothesis! Such the spacelike interval has been excluded. LT naturally has the restriction that any velocity cannot be larger than the speed of light c.
When we give up this hypothesis, then besides (4), another symmetrical system exists yes. So long as the simultaneity t'=0 holds, from (3) $th\phi = x_0/x_1 = c/\bar{v}$ is obtained, then we derive necessarily the generalized Lorentz transformation (GLT) [24, 13]:

$$x_1{}' = \bar{\gamma}(x_1 - c^2t/\bar{v}), t' = \bar{\gamma}(t - x_1/\bar{v}), \tag{5}$$

where $\bar{\gamma} = 1/\sqrt{1-(c/\bar{v})^2}$. LT and GLT are connected by the well-known de Broglie relation $v\bar{v} = c^2$, in which v is group velocity and soliton velocity, and $\bar{v}$ is phase velocity, which must be superluminal.
The superluminal GLT may solve many problems on $\Delta t \to 0$ and the simultaneity. Therefore, we must investigate the spacelike interval, and corresponding superluminal.

3. Contradictions between Quantum Theory and Special Relativity

Usual physicists believe that quantum theory and special relativity are unified completely by Klein-Gordon equation and Dirac equation, etc. But, we first proposed that the constancy of the vacuum speed of light in relativity is contradiction with the uncertainty principle of quantum mechanics. Further, we obtained the uncertainty relations of usual velocity and the speed of light, and which should have the statistical fluctuations [25].
Based on quantum mechanics we obtained the uncertainty relations of velocity as a ratio of position to time are:

$$\Delta v \Delta(pt) = \hbar, \tag{6}$$

or

$$\Delta v \Delta(mx) = \hbar. \tag{7}$$

The speed of light as the velocity is also uncertain, for v=c (6)(7) become:

$$\Delta c\Delta(\frac{\nu}{c^2}x) = 1, \quad (8)$$

or

$$\Delta c\Delta(\frac{\nu}{c}t) = 1, \quad (9)$$

where ν is the frequency of light, x and t are position and time of uncertainty, respectively. So long as the uncertainty principle holds, the speed of light possesses uncertainty necessarily, moreover, when x and t and ν are made more definite, c becomes less definite. The speed of light should have the statistical fluctuations inside a small space-time and at high energy. Further, some fundamental constants probably are also uncertain. We proposed that relativity and quantum theory will be able to be unified completely only after both are developed [25, 26].

Further, based on the uncertainty principle in quantum theory and the generalized uncertainty principle, we researched uncertain relations of entropy, information and thermodynamics, and proposed the speed of light and some fundamental constants should be uncertain. We searched also its possible development in the entanglement, superstring, and the extensive quantum theory, various matters and different phases, etc. Moreover, the irreversibility and statistics are discussed, and we derive operators of entropy, etc. New measuring technology and some developed theories already challenge the uncertainty principle, which may not be held under certain conditions, and must be modified and developed [27].

Generally, if the uncertainty principle holds always, separable particles should be come to an end. The known size of superstring is very small $\Delta x \cong 10^{-35} cm$. According to the uncertainty principle, a corresponding momentum of superstring is:

$$\Delta p = h/\Delta x \cong 6.626 \times 10^8 gcm/s. \quad (10)$$

If the velocity of superstring is approximately velocity of light, its moving-mass will possess a macroscopic mass $5.618 \times 10^{26} MeV/c^2 = 2.209 \times 10^{-2} g$ [26].

Next, according to the uncertainty principle in quantum theory, the same point △x=0 and the simultaneous △t=0 all are not possible. According to the special relativity, the same point △x=0 in the timelike interval and the simultaneous △t=0 in the spacelike interval cannot hold simultaneously. So strict timelike and spacelike are not valid. They are uncertainties, and have fuzzy boundaries.

In the double-slit experiments, without the detector, each electron passes simultaneously through two slits, showing a wave; when the detector is placed in a narrow slit, the wave disappears, and particles appear, and this is the same place. Further, it can show the ambiguity of confused indistinct gray shadows that are not duality either white or black.

For the open setting the available particle is observed separately, while for the closed setting the wave of the available interference display is observed simultaneously. Experiments seem to determine that two aspects correspond to the same spot and timelike, and simultaneous and spacelike, respectively. This is a new interpretation of the duality in the double-slit experiment.

Generally, the particle property is the determination of the same position, and corresponds to the timelike interval, such as photoelectric effect, Compton effect, and so on. The wave property is determined simultaneously, and corresponds to the spacelike interval [28]. The interference pattern of wave is the result of simultaneously observing multiple scattering. The two are mutually exclusive and complementary.

Probably, the spacelike and the timelike intervals correspond to the wave-particle duality.

The distribution of the whole probability is observed simultaneously, it shows the form of a wave. The probability waves correspond to between the phases, and to GLT.

In quantum mechanics the uncertainty restricts the very small regions. Special relativity restricts the spacelike interval, and repulses superluminal. There are contradictions between the two aspects.

The same point corresponds to the timelike interval. Time cannot be reversed, directional, inside the light cone, v<c. There has the causality. The simultaneous time corresponds to the spacelike interval. Space can be reversed, without directionality, outside the light cone, v>c. There has not the causality, and has wormholes and time travel. The two aspects of space and time are correspondingly opposite.

In the uncertainty relations, △t is smaller, which reaches to the spacelike interval, and △E is larger. The smaller △x reaches timelike interval, and △p is lager (*i.e.*, the instantaneous speed). This indicates that the timelike and spacelike intervals are mutually exclusive. It is consistent with special relativity.

The entanglement is the non-locality on space. The delayed-choice experiments are the non-locality on time [29].

Wave packet collapse (*i.e.*, decoherence) occurs transiently during observation, which correspond to simultaneity, the spacelike interval and superluminal v>0.

Vibration is at the same place for different time; wave is at different places for the same time. The propagation of vibrations forms wave. Particles, from nucleons, electrons to the more general "quantum" all have various vibrations, motion and Brownian motion, etc. Their propagation forms wave. The nebular motion produces the celestial quantum mechanics and the corresponding Schrödinger quantum equation [30], etc.

Simple or simplified harmonic vibrations and waves have the invariant period T and frequency v=1/T. The same motion corresponds to H is the same, v larger and energy E=Hv larger. When motion propagates in space, speed and wavelengths are introduced.

General wave is:

$$y = A\cos[v(t - \frac{x}{c})] \to Ae^{i(vt - x/\lambda)} = Ae^{ik_i x_i}. \tag{11}$$

So $k_i = i\frac{\partial}{\partial x_i}$ is an operator. Further, we may derive the wave equations without h. Combining the Planck and de Broglie formulas $p_i = \hbar k_i$ which correspond to the duality, quantum mechanics is obtained. If it is extended to $p_i = Hk_i$, we obtain the duality and basic equations in the extensive quantum theory [30]. It corresponds to timelike and spacelike intervals.

Based on the non-commutation $[A, B] = AB - BA = \eta \neq 0$ of matrix, group, tensor and so on, we proposed the mathematical quantum theory. If η is imaginary number, it will correspond to the extensive quantum theory. If η is real number, it will be development of quantum theory. Moreover, η may be complex number, etc. We introduced a similar wave function and corresponding operators, various similar quantum results will be derived. Further, we discussed its physical meaning and various applications. Based the general matrix we researched mathematics of unified gravitational and electromagnetic fields, and discussed the space-time equations and the simplest unifying quantum theory and general relativity [31].

Based on quantum entanglement and corresponding quantum communication, we researched a simple superluminal entangled communication scheme, whose key is to establish two mutually entangled particles or devices A and B. When we observe and control the information of A position, then can know the corresponding results of the other B. This is not to send directly information each other [32]. It may be superluminal. In special relativity we provided that there are necessarily two symmetrical topological structures separated by the light-cone, which includes GLT for the spacelike interval, in which phase velocity is superluminal [24, 13]. It is base of this scheme, and may test GLT.

Uncertainty is an important principle in quantum theory [26, 33], and is also a characteristic of any wave. So waves should also have various uncertain properties, such as the relation between energy and time, etc.

In simplified Newtonian mechanics and Schrödinger equation, the potential energy V becomes to gravity, and three other interactions. Further, the unity of quantum mechanics and general relativity will be that the vibration in curved space-time develops to wave. Geometrically this is space-time discontinuity, the light-cone, c and the timelike and spacelike discontinuity.

The wave function $\psi(r,t)$ indicates that the particle occupies simultaneously the entire space r at the same time t. The wave function can describe the quantum system only at the moment of the measurement. Particles in quantum mechanics cannot be found at the same time and at the same place. The same is true of the more general physical functions f(r,t).

4. Unified Quantum Theory and Complete Special Relativity

We should research LT for very small $\triangle r$ or $\triangle r \to 0$, and GLT for very small $\triangle t$ or $\triangle t \to 0$.

For (2), if x cannot be 0, and $\triangle x$ is very small,

$$\Delta x = x_1 ch\phi - x_0 sh\phi, \tag{12}$$

we obtain

$$th\phi = (x_1/x_0) - (\Delta x/ctch\phi) \approx (v/c) - (\Delta v/c). \tag{13}$$

Then we derive the new Lorentz transformation (NLT):

$$x_1{}' = \gamma_{\Delta x}[x_1 - (v - \Delta v)t], \tag{14}$$

$$t' = \gamma_{\Delta x}[t - (v - \Delta v)x_1/c^2], \tag{15}$$

where $\gamma_{\Delta x} = 1/\sqrt{1 - [(v - \Delta v)/c]^2}$.

By using the same method, for (3) if t cannot be 0, and $\triangle t$ is very small,

$$c\Delta t = x_0 ch\phi - x_1 sh\phi, \tag{16}$$

we obtain

$$th\phi = (x_0/x_1) - (c\Delta t/x_1 ch\phi) \approx \frac{c}{\bar{v} - (x_1/\Delta t)}. \tag{17}$$

Then we derive the new generalized Lorentz transformation (NGLT):

$$x_1{}' = \bar{\gamma}_{\Delta t}[x_1 - c^2 t/(\bar{v} - x_1/\Delta t)], \tag{18}$$
$$t' = \bar{\gamma}_{\Delta t}[t - x_1/(\bar{v} - x_1/\Delta t)], \tag{19}$$

where $\bar{\gamma}_{\Delta t} = 1/\sqrt{1 - [c/(\bar{v} - x_1/\Delta t)]^2}$.

In this case, their uncertainty is:

$$c\Delta x\Delta t = x_1x_0(ch^2\phi + sh^2\phi) - (x_1^2 + x_0^2)ch\phi sh\phi. \quad (20)$$

In quantum mechanics

$$\Delta x\Delta t = h^2/\Delta p\Delta E. \quad (21)$$

For any four-vector $(\vec{A}; A_0)$, its LT is

$$A_1' = \gamma(A_1 - vA_0/c), A_0' = \gamma(A_0 - vA_1/c), \quad (22)$$

and GLT is

$$A_1' = \bar{\gamma}(A_1 - cA_0/\bar{v}), A_0' = \bar{\gamma}(A_0 - cA_1/\bar{v}). \quad (23)$$

Both possess the most perfect symmetrical form. Only A_1, A_0 interchange each other between A_1 and A_0 representations, and LT and GLT also interchange from v/c to c/$\bar{v}$.

For a four-vector (P; E), LT is

$$P_1' = \gamma(P_1 - vE/c^2), E' = \gamma(E - vP_1), \quad (24)$$

and GLT is

$$P_1' = \bar{\gamma}(P_1 - E/\bar{v}), E' = \bar{\gamma}(E - c^2P_1/\bar{v}). \quad (25)$$

5. Discussion

From 1999 to 2022, some experiments with very smal speed of light opposite to superluminal were also confirmed [34-38]. First, Hau, *et al*., found light speed reduction to 17 metres per second in an ultracold atomic gas [34]. It is a great development of Cerenkov radiation. Bajcsy, *et al*., observed coherent optical information storage in an atomic medium using halted light pulses [35]. Boyd, *et al*., discussed the controlling the velocity of light pulses [36]. Safavi-Naeini, *et al*., studied electromagnetically induced transparency and slow light with optomechanics [37]. Mikaeili, *et al*., reported ultraslow light realization using an interacting Bose-Einstein condensate trapped in a shallow optical lattice [38].

The general wave has the uncertainty relation of wave. For this we must also observe a certain area simultaneously, otherwise it is still a vibration at this point.

Further, this is extended to all waves, and corresponds to simultaneity and spacelike interval. But, usual wave corresponds to a group speed, and is not superluminal. In the extensive special relativity it can be generalized to general invariant speed c_h [24, 23], and super-c_h, and the c_h-cone and corresponding time-like and space-like intervals.

Wave must generally be measured simultaneously for a certain range. When they propagate to long distances, the relation between each other can be superluminal, and belong to spacelike interval.

Evolution and change are all to begin from a certain moment, and to a later moment. This is all in the spacelike interval. Each section is a spacelike interval. They are not necessarily related to each other, during which the speed can be superluminal.

K-G equation and Dirac equations are independent of LT. For various 4-vectors, the spacelike and timelike intervals can be developed in parallel. Various quantum theories related to LT can be developed to theories with GLT, NLT, NGLT and their operators and related forms, including operators, equations, commutation relations, etc.

The timelike and spacelike relations in P, T. C, PC, etc., are associated with space and time, and are related to their operators. These should be investigated.

6. References

1. Pearson TE, Unwin SC, Cohen MH, Linfield RP, Readhead AC, Seielstad GA, Simon RS, Walker RC. Superluminal expansion of quasar 3C273. Nature. 1981 Apr;290(5805):365-8-368.
2. Rees MJ. Appearance of relativistically expanding radio sources. Nature. 1966 Jul;211(5048):468-70.
3. Blandford RD, Königl A. Relativistic jets as compact radio sources. The Astrophysical Journal. 1979 Aug;232:34-48.
4. Vermeulen RC. Int. Astron. Union Symp. 1996;175:57.
5. Scharnhorst K. On propagation of light in the vacuum between plates. Physics Letters B. 1990 Feb 22;236(3):354-359.
6. Barton G. Faster-than-c light between parallel mirrors The Scharnhorst effect rederived. Physics Letters B. 1990 Mar 22;237(3-4):559-562.
7. Steinberg AM, Kwiat PG, Chiao RY. Measurement of the single-photon tunneling time. Physical Review Letters. 1993 Aug 2;71(5):708.
8. Steiberg AM, Chiao RY. Phys. Rev A. 1994;49(3):2071.
9. Aharonov Y, Reznik B, Stern A. Phys. Rev. Lett. 1998;81(11):2190.
10. Mugnai D, Ranfagni A, Ruggeri R. Observation of superluminal behaviors in wave propagation. Physical review letters. 2000 May 22;84(21):4830.
11. Wang LJ, Kuzmich A, Dogariu A. Gain-assisted superluminal light propagation. Nature. 2000 Jul;406(6793):277-279.

12. Yi-Fang Chang. Extension and complete structure of the special relativity included superluminal and neutrino-photon with mass. International Journal of Modern Theoretical Physics. 2013:2(2):53-73.
13. Kai Wang, Shao-Qiang Xi, Lijing Shao, Ruo-Yu Liu, Zhuo Li, Zhong-Kai Zhang. Limiting superluminal neutrino velocity and Lorentz invariance violation by neutrino emission from the blazar TXS 0506+056. Phys. Rev. D. 2020;102(6):063027.
14. Gusikhin PA, Muravev VM, Kukushkin IV. Superluminal electromagnetic two-dimensional plasma waves. Phys. Rev B. 2020;102(12):121404(R).
15. Kazuhiro Morimoto, Ming-Lo Wu, Andrei Ardelean, Edoardo Charbos. Superluminal motion- assisted four-dimensional light-in-flight imaging. Phys. Rev X. 2021;11(1):011005.
16. Tasgin ME. Negative superluminal velocity and violation of Kramers-Kronig relations in causal optical systems. Phys. Rev A. 2021;103(1):013504.
17. de Ramón J, Papageorgiou M, Martín-Martínez E. Relativistic causality in particle detector models: Faster-than-light signaling and impossible measurements. Phys. Rev D. 2021;103(8):085002.
18. Tong Xi, Wang Yi, Yuhang Zhu. Bit rate bound on superluminal communication. Phys. Rev D. 2021;104(8):084062.
19. Li F, Dalichaouch T N, Pierce JR, Xu X, Tsung FS, Lu W, *et al*. Ultrabright electron bunch injection in a plasma Wakefield driven by a superluminal flying focus electron beam. Phys. Rev. Lett. 2022;128(77):174803.
20. Qianqian Chen, Yaoming Chu, Jianming Cai. Simulating superluminal propagation of Dirac particles using trapped ions. Phys. Rev A. 2022;105(4):042609.
21. Apostolakis A, Balanov AG, Kusmartsev FV, Alekseev KN. Beyond the ordinary acoustoelectric effect: Superluminal phenomena in the acoustic realm and phonon-mediated Bloch gain. Phys. Rev B. 2022;106:014312.
22. Vilasini V, Colbeck R. Impossibility of superluminal signaling in Minkowski spacetime does not rule out causal loops. Phys. Rev. Lett. 2022;129(11):110401.
23. Yi-Fang Chang. New Research of Particle Physics and Relativity. Yunnan Science and Technology Press. 1989. Phys. Abst. 1990;93:1371.
24. Yi-Fang Chang. Contradiction between the uncertainty principle and the Constancy light speed. Galilean Electrodynamics. 2001;12(4):77-79.
25. Yi-Fang Chang. Uncertain physics, fluctuation-chaos and general uncertain sciences. International Journal of Modern Mathematical Sciences. 2016;14(4):386-397.
26. Yi-Fang Chang, Hadronic J. 2021;44:357.
27. Arndt M, Nairz O, Vos-Andreae J, Keller C, Van der Zouw G, Zeilinger A. Wave–particle duality of C60 molecules. nature. 1999 Oct;401(6754):680-682.
28. Bahrami M, Shafiee A. Postponing the past: An operational analysis of delayed-choice experiments. Foundations of Physics. 2010;40(1):55-92.
29. Yi-Fang Chang. Development of Titius-Bode law and the extensive quantum theory. Physics Essays. 2002;15(2):133-137.
30. Yi-Fang Chang. Matrix, group, tensor, etc., and corresponding mathematical quantum theory and its applications. Hadronic Journal. 2021;44(3):251-267.
31. Yi-Fang Chang. Simple superluminal entangled communication scheme and its base of special relativity. Asian Journal of Research and Reviews in Physics. 2021;5(3):34-40.
32. Yi-Fang Chang. New research and development of uncertainty relations. Hadronic Journal. 2021;44(4):357-378.
33. Hau LV, Harris SE, Dutton Z, Behroozi CH. Light speed reduction to 17 metres per second in an ultracold atomic gas. Nature. 1999;397(3720):594-598.
34. Bajcsy M, Zibrov AS, Lukin MD. Stationary pulses of light in an atomic medium. Nature. 2003;426(6967):638-641.
35. Boyd RW, Gauthier DJ. Controlling the velocity of light pulses. Science. 2009;326(5956):1074-1077.
36. Safavi-Naeini AH, Alegre TPM, Chan J, *et al*. Electromagnetically induced transparency and slow light with optomechanics. Nature. 2011;472(7341):69-73.
37. Mikaeili H, Dalafi A, Ghanaatshoar M, Askari B. Ultraslow light realization using an interacting Bose-Einstein condensate trapped in a shallow optical lattice. Scientific Reports. 2022;12(1):4428.