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Wave number and magnetic field effects on the boundary layer flow over an accelerating vertical surface with zero-time periodic boundaries

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Abstract
The problem of the wave number and magnetic field effects on a boundary layer flow over an accelerating vertical surface in the presence of heat generation and reacting species, and zero-time periodic boundaries is examined. We assumed the fluid is Newtonian, chemically reactive and of order one, and magnetically susceptible. The governing non-linear partial differential equations of the Bousinesq form are solved using the Homotopy Perturbation Method. Expressions for the concentration, temperature, and velocity are obtained, computed, and presented graphically. The analysis of results shows that the increase in the: Wave number increases the concentration, temperature, velocity, Sherwood, Nusselt number, and stress on the wall; Magnetic field increases the concentration, temperature, and velocity in the range of $0 < M < 1.5$; Dufour number increases the concentration, temperature, and velocity of the fluid.

Keywords: Boundary layer, heat and mass transfer, hydro-magnetic, non-time periodic boundaries, vertical surface

Introduction

Just as free convection has applications in many engineering fields, especially in automatic control systems, and periodic heating and cooling processes; MHD heat, and mass transfer have relevance in the aerodynamics extraction of plastic sheets, energy storage units, biological transportation, liquid metal fluids, oil reservoirs, high plasmas, geothermal systems, heat insulation, and metal and polymer extrusion, thermal energy devices, and the like.

The nature of the boundary conditions of any fluid dynamical problem determines the nature of its solutions. Based on this, a lot of studies have been carried out on the flow over moving flat plates alongside diverse boundary conditions. Some considered the cases where the boundary conditions are constant and others where they vary with some dependent variables. On the varying boundary case, some considered the case where they are regular, some where they are slope or step functions/ramped, some where they are period/oscillatory, some where the thermal and mass diffusion are convective, and others with slip velocity. Our review of literature in this paper shall be alongside boundary conditions. In the cases with variable boundaries, the flows over a plate with convective boundaries have been studied. For example, Aziz [1] studied the laminar case using the method of similarity transformation; Ajadi *et al.* [2] examined the non-Newtonian case with slip velocity; Makinde [3] studied numerically the mixed convective flow using similarity solution, and noticed that the Nusselt number and wall shear stress increase with the increase in the magnetic field, buoyancy force and local convective heat exchange parameters. Makinde [4] looked at the heat and mass effects on the MHD fluid flow; Makinde and Olarewaju [5] considered the buoyancy effects on the thermal boundary layer flow; Makinde [6] studied the effects of internal heat generation using similarity transformation method of solution; Gangadkar *et al.* [7] gave the analysis of the flow using similarity transformation. Rout *et al.*, [8] investigated the transient MHD chemically reacting flow with heat source effects using similarity transformation and fourth-order Runge-Kulta numerical approaches; Abbasi *et al.* [9] examined the hydrodynamic and thermal boundary layers flow using an analytic approach; Emmanuel *et al.* [10] worked on the effects of thermal radiation, viscous dissipation on the Newtonian fluid flow.

Eckert number increases the skin friction, plate surface temperature, and thermal and velocity boundary layer but decreases the local Nusselt number; an increase in the Biot number increases the plate temperature and local Nusselt number; increase in the Hartmann number increases the wall shear stress, plate surface temperature, thermal and velocity boundary layer thicknesses but decreases the local Nusselt number; Imoro *et al*. [12] looked at the flow in the presence of viscous dissipation and nth order chemical reaction.

Similarly, for regular boundaries, Yoo [13] studied the flow with spatially periodic boundary temperature using regular perturbation series expansion; Raju and Varma [14] studied hydro-magnetic natural convective oscillatory flow with periodic wall temperature.

More so, for cases with step-functions, Chandram *et al*. [15] examined the free convection near a vertical plate with ramped wall temperature; Seth *et al*. [16] studied the effects of rotation on the MHD free convective flow over an impulsively accelerating vertical plate with ramped temperature; Rajesh [17] considered the effects of chemical reaction and radiation on the unsteady hydro-magnetic natural convective flow of dissipative fluid over a vertical plate with ramped wall temperature; Das *et al*. [18] investigated the unsteady MHD flow of heat absorbing dusty plasma fluid over a porous vertical plate with ramped temperature. Khan *et al*. [19] examined the roles of skin friction on the transient hydro-magnetic conjugate flow in a porous medium with ramped wall temperature; Rams and Raju [50] considered the MHD heat and mass transfer characteristics on Newtonian fluid past a vertical permeable plate with ramped boundary condition; Sinha *et al*. [21] looked into the natural convective flow past a vertical plate with ramped wall temperature.

Furthermore, studies have shown that at solid walls, the boundary condition for a viscous fluid obeys no-slip: the fluid velocity takes the velocity of the solid boundary. However, at very high altitude fluid particles adjacent to the surfaces of aircraft and rockets no longer takes the velocity of the surface but possesses a finite tangential velocity known as the slip velocity, which slips along the surface. This ideology was first conceived by Navier [22] who presented a slip flow model, where he suggested the fluid velocity at the plate to be linearly proportional to the shear stress at the plate. Building on this, Maxwell [23] formulated a slip model, which is being used extensively by researchers up to date. Upon Maxwell’s model, a lot of research had been carried out. For example, Sparrow *et al*. [24] studied the slip flow at the entrance region of a parallel plate channel; Eber and Sparrow [25] considered the flow in rectangular and annular ducts; Sharma and Chaudhary [26] investigated the effects of variable suction on the unsteady natural convective flow of viscous incompressible flow over a vertical plate with periodic temperature in the presence of slip condition; Singh and Gupta [27] examined the MHD natural convective slip flow of viscous fluid through an oscillating porous plate. Furthermore, Singh *et al*. [28] studied the MHD free convective flow of viscous fluid over a porous vertical plate through a non-homogeneous porous medium with radiation and temperature gradient-dependent heat source in a slip flow regime; Mohmood and Ali [49] considered the effect of slip condition on a transient MHD oscillatory flow of a viscous fluid in a planer channel; Devi and Raj [30] looked at the effects thermo-diffusion on the transient MHD natural convective flow a moving vertical plate in the presence of time-dependent suction, heat source and slip velocity. Baoku *et al*. [31] considered the MHD flow of Third-grade fluid over an insulated plate with heat and mass transfer and slip effects; Sengupta and Ahmed [32] studied the MHD natural convective chemically reactive flow over a vertical plate in a velocity flow regime with thermo-diffusion, fluctuating wall temperature and concentration, and thermal radiation, and free stream velocity; Adesanya and Makinde [33] examined the MHD oscillatory slip flow and heat transfer in a channel filled with porous media. Seini and Makinde [34] considered the MHD boundary layer flow with slip near a stagnation point on a vertical plate; Kumar *et al*. [35] investigated thermal diffusion and chemical reaction effects on an unsteady flow past a vertical porous plate in the presence of temperature-dependent heat source and slip condition; Kharabela *et al*. [36] examined numerically the higher-order chemical reaction effects on MHD Nano-fluid flow with slip boundary condition.

In another development, flow problems over moving vertical plates with constant boundary conditions have been looked into as independent variables. For example, Devi *et al*. [37] studied the boundary layer flow over a wedge with chemical reaction, heat and mass transfer, and suction/injection effects; Kandasamy *et al*. [38] examined the MHD flow past a vertical stretching sheet with chemical reaction, heat source, and thermal reaction stratification effects; Ibrahim *et al*. [39] investigated the unsteady MHD free convective flow over a porous vertical plate with chemical reaction, heat source, and suction effects. Mohammed and Abo-Dahab [40] studied the MHD flow of micro-polar fluid over a porous vertical plate in the presence of chemical reaction, thermal radiation, and heat generation; Ramanchandra *et al*. [41] examined the MHD fluid flow over a porous vertical plate in a non-Darcian porous medium with cross-diffusion effects. Khan *et al*. [42] considered the unsteady MHD mixed convective flow over a porous vertical plate with heat generation effects; Murthy [43] investigated the MHD natural convective flow over a porous vertical plate with heat and mass transfer, thermal radiation and Hall effects; Reddy *et al*. [44] studied the MHD free convective flow over a vertical plate with heat and mass transfer, chemical reaction and radiation absorption effects. Islam *et al*. [45] investigated the unsteady MHD free convective boundary layer flow over a vertical plate in the presence of chemical reaction, radiation, and heat absorption.

Based on the above research reports, this paper aims to study the effects of Wave number and magnetic field on a steady boundary layer flow over an accelerating vertical surface with zero-time periodic boundaries using Homotopy Perturbation Method.

Materials and Method

Physics of Problem and Mathematical Formulation

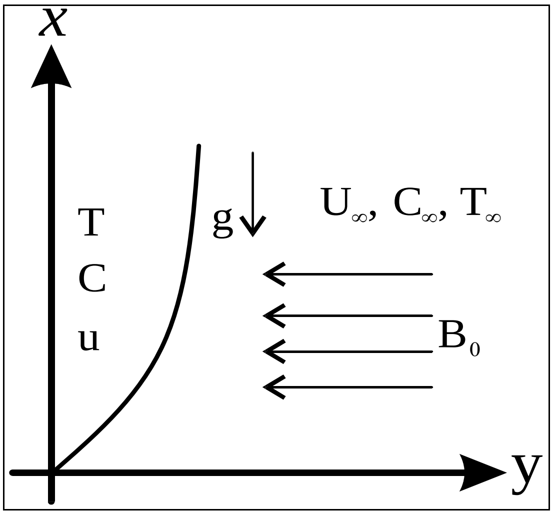

Fig 1: The model of an oscillatory vertically accelerating plate in a fluid

We consider a 2-D transient hydro-magnetic flow past an oscillatory moving vertical plate with a heat source, viscous dissipation, and periodic surface boundary effects. The model is formulated on the following premises: the fluid is electro-magnetically conducting and chemically reactive, and the physical properties of the fluid remain constant. The plate is not heated to a high-temperature regime; therefore thermal radiation is absent. The fluid is mixed with a chemical species at a higher concentration to initiate a chemical reaction. The plate is infinite and the Darcy-type porosity. A transverse magnetic field of uniform strength and negligible induction effect is applied. In the model, the x -axis is taken to be in the vertical direction of the plate and the y-axis is normal to it. Therefore, if (u, v) are the velocity components in the spatial (x, y, t) coordinates; T and C are the fluid temperature and concentration and T_w $C_w$are the temperature and concentration at the wall at $t > 0$; T_∞ andC_∞ are the equilibrium temperature and concentration at $t = 0$, then the governing equation of continuity, momentum, energy, and diffusion are

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \tag{1}$$

$$\rho\left(u'\frac{\partial u'}{\partial x'} + v'\frac{\partial u'}{\partial y'}\right) = \mu\frac{\partial^2 u'}{\partial y'^2} + \rho g\beta_t(T' - T_\infty) + \rho g\beta_c(C' - C_\infty) - \left(\frac{\sigma_e {B_o}^2}{\mu\mu_m} - \frac{\mu}{\kappa}\right)u' \tag{2}$$

$$\rho C_p\left(u\frac{dT'}{dx'} + v\frac{\partial T'}{\partial y'}\right) = k_o\frac{\partial^2 T'}{\partial y'^2} + Q(T' - T_\infty) + \mu\left(\frac{\partial u'}{\partial y'}\right)^2 + \frac{Dk_T}{C_s\rho C_p}\frac{\partial^2 C}{\partial y^2} \tag{3}$$

$$u'\frac{\partial C'}{\partial x'} + v'\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} - {k_r}^2(C' - C_\infty) + \frac{Dk_T}{T_m}\frac{\partial^2 T'}{\partial y'^2} \tag{4}$$

where μ is the dynamic viscosity; ρ is the density; g is the acceleration due to gravity; β_t is the volumetric coefficient of expansion due to temperature; β_c is the volumetric coefficient of expansion due to concentration; k_o is the thermal conductivity of the fluid; μ_m is the magnetic field permittivity; Q is the heat source/sink; σ_e is the electrical conductivity of the fluid; B_o^2 is the magnetic field flux;C_p is the specific heat capacity at constant pressure; D is the co-efficient of mass transfer/diffusivity, κ is the permittivity of the porous plate; k_r^2 is the chemical reaction rate of the species concentration.

Mathematical Foundation for the Flow Model

Waves are generated by vibrating or agitated objects, irrespective of where they are. A typical mechanical wave requires a medium to travel. Consider a moving plate in a fluid. At $t' = 0$ both the plate and the fluid are at rest with uniform temperature and concentration and as such that waves are absent. At $t' > 0$, the plate begins to oscillate in its plane$(y' = 0)$. By the principle of super-imposition, both the plate and the fluid are oscillating, therefore the resultant wave is the sum of the waves, and their displacement is prescribed as $Y(x', t') = ASin(\omega t' + kx') + BCos(\omega t' + kx' + \phi)$, where$A$and $B$are the amplitudes (maximum heights of the wave from the mean position);$\omega = \frac{2\pi}{T} = 2\pi f$is the angular velocity of the oscillating plate, and $f$is the frequency of the oscillation, $T = 2\pi\sqrt{\frac{l}{g}}$is the period of the wave (time for one-to-and fro oscillation),$l$is the characteristic length of the plate,$k = \frac{2\pi}{\lambda}$is the wave number (Womersley number), which may be expressed in different forms non-dimensionally; $\lambda$is the length of the wave crest or trough, $x'$is the distance between successive crests or troughs, $ASin(\omega t' + kx')$ is the oscillating term, Cos $(\omega t'+kx'+\phi)$ is the phase, and $\phi=\frac{2\pi x'}{\lambda}$is the lead angle or phase constant. The strength of a wave may be more pronounced in the direction of an inducing force.

Generally, trigonometric functions are periodic (Spiegel) [46]. Therefore, waves are periodic. Wave displacements may take several forms. Neglecting the phase angle, taking the wave to be effective in the upwards direction, the oscillating wave can simply be prescribed for the time-periodic as $Y(x',t')=Sin(\omega t'+kx')=e^{i\omega t'}$ or, $Y(x',t')=Cos(\omega t'+kx')=e^{i\omega t\prime}$ (see Adesanya *et al.*) [47], Similarly, for a zero-time periodic, we write $Y(x',0')=Sin(kx')=e^{ikx'}$ or, $Y(x',0')=Cos(kx')=e^{ikx'}$

Upon the foregoing, the boundary conditions are chosen as

$$t\le 0: u'=0, v'=0, T'\to T_\infty, C'\to C_\infty \text{ everyawhere}$$

$$t>0: u'=\gamma U_o, v'=0, T'=T_w=exp(\,ikx'), C'=C_w=exp(\,ikx') \text{ at } y=0 \tag{5}$$

$$u'=0, v'=0, T'\to T_\infty, C'\to C_\infty at\ \ y\to\infty \tag{6}$$

where U_o is the characteristic velocity of fluid/plate, and $\gamma=1$for accelerating plate (Azziz *et al.*, 2009) [2].

Introducing the following dimensionless quantities:

$$x=\frac{x'U_o}{\upsilon}, y=\frac{y'U_o}{\upsilon}, x=\frac{x'U_o}{\upsilon}, t=\frac{t'U_o}{\upsilon}, u=\frac{u'}{U_o}, v=\frac{v'}{U_o}, \Theta=\frac{T-T_\infty}{T_w-T_\infty}, \Phi=\frac{C-C_\infty}{C_w-C_\infty},$$

$$Gr=\frac{\rho g\beta_t(T_w-T_\infty)}{U_o^2}, Gc=\frac{\rho g\beta_c(C_w-C_\infty)}{U_o^2}, Ec=\frac{U_o^2}{kC_p(T_f-T_\infty)},$$

$$M^2=\frac{\sigma_e B_o^2\upsilon}{\mu\mu_m U_o^2}, \chi^2=\frac{\upsilon}{\kappa}, Pr=\frac{\upsilon}{k}, Sc=\frac{\upsilon}{D_m}, \delta^2=\frac{k_r^2}{D_m}, N^2=\frac{Q}{\rho kC_pU_o}, Sr=\frac{Dk_T(T_2-T_1)}{T_m(C_2-C_1)}, Dr=\frac{Dk_T(C_2-C_1)}{C_s(T_2-T_1)} \tag{7}$$

(whereΘ and Φ are the non-dimensionalized temperature and concentration, respectively;$N$is the temperature difference parameter; Gr are the Grashof number due to temperature difference;Ec is the Eckert number; M^2 is the magnetic field force χ^2 is the porosity parameter; Pr is the Prandtl number; Sc is the Schmidt number;$\delta^2$is the chemical reaction rate, k_T is the diffusivity ratio of the fluid, $T_m$is the mean temperature of the fluid, $C_s$is the concentration susceptibility of the fluid, $Sr$is the Soret number, $Dr$is the Dufour number) into equations (1)-(6), we have

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{8}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}-M_1^2u+Gr\Theta+Gc\Phi \tag{9}$$

$$u\frac{\partial\Theta}{\partial x}+v\frac{\partial\Theta}{\partial y}=\frac{\partial^2\Theta}{\partial y^2}+N^2Pr\,\Theta-Ec\left(\frac{\partial u}{\partial y}\right)^2-Dr\Phi'' \tag{10}$$

$$u\frac{\partial\Phi}{\partial x}+v\frac{\partial\Phi}{\partial y}=\frac{\partial^2\Phi}{\partial y^2}-Sc\delta^2\Phi-Sr\Theta' \tag{11}$$

with the boundary conditions

$$u=1, v=0, \Theta=exp(\,ikx'), \Phi=exp(ikx') \text{ at } y=0 \tag{12}$$

$$u=0, v=0, \Theta=0, \Phi=0 \text{ at } y=\infty \tag{13}$$

Method of Solution

Equations (16) - (20) are strongly coupled, and as such, we seek the Homotopy Perturbation Method

N(u)+L(u)-f(t)=0

to make them tractable. To achieve this, we re-arrange equations (9) - (11) as

$$\frac{\partial^2 u}{\partial y^2} = u\frac{\partial u}{\partial y} + v\frac{\partial u}{\partial y} + Mu - Gr\Theta - Gc\Phi \tag{14}$$

$$\frac{\partial^2 \Theta}{\partial y^2} = u\frac{\partial \Theta}{\partial y} + v\frac{\partial \Theta}{\partial y} - N\,Pr\,\Theta + Ec\left(\frac{\partial u}{\partial y}\right)^2 + Dr\Phi'' \tag{15}$$

$$\frac{\partial^2 \Phi}{\partial y^2} = u\frac{\partial \Phi}{\partial y} + v\frac{\partial \Phi}{\partial y} + Sc\delta\Phi + Sr\Theta' \tag{16}$$

with the boundary conditions, equations (12) – (13) still holding

Expressing these in the Homotopy Perturbation arrangement, we have

$$(1-p)\frac{\partial^2 u}{\partial y^2} = p\left(-\frac{\partial^2 u}{\partial y^2} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + Mu - Gr\Theta - Gc\Phi\right) \tag{17}$$

$$(1-p)\frac{\partial^2 \Theta}{\partial y^2} = p\left(-\frac{\partial^2 \Theta}{\partial y^2} + u\frac{\partial \Theta}{\partial x} + v\frac{\partial \Theta}{\partial y} - N\,Pr\,\Theta + Ec\left(\frac{\partial u}{\partial y}\right)^2 + Dr\Phi''\right) \tag{18}$$

$$(1-p)\frac{\partial^2 \Phi}{\partial y^2} = p\left(-\frac{\partial^2 \Phi}{\partial y^2} + u\frac{\partial \Phi}{\partial x} + v\frac{\partial \Phi}{\partial y} + Sc\delta\Phi + Sr\Theta''\right) \tag{19}$$

Expressing the dependent variables series expansions of p, we have

$$\left.\begin{array}{l} u = u_o + pu_1 + p^2u_2 + p^3u_3 + \ldots \\ \Theta = \Theta_o + p\Theta_1 + p^2\Theta_2 + p^3\Theta_3 + \ldots \\ \Phi = \Phi_o + p\Phi_1 + p^2\Phi_2 + .p^3{}_3\Phi + .. \end{array}\right\} \tag{20}$$

Substituting equation (20) into equations (17) - (19) gives

$$\frac{\partial^2 u_o}{\partial y^2} + p\frac{\partial^2 u_1}{\partial y^2} + p^2\frac{\partial^2 u_2}{\partial y^2} + p^3\frac{\partial^2 u_3}{\partial y^2} + .. = p\begin{bmatrix} u_o\frac{\partial u_o}{\partial x} + p\left(u_o\frac{\partial u_1}{\partial x} + u_1\frac{\partial u_o}{\partial x}\right) + p^2\left(u_o\frac{\partial u_2}{\partial x} + u_1\frac{\partial u_1}{\partial x} + u_2\frac{\partial u_o}{\partial x} + ..\right) \\ v_o\frac{\partial u_o}{\partial y} + p\left(v_o\frac{\partial u_1}{\partial y} + v_1\frac{\partial u_o}{\partial y}\right) + p^2\left(v_o\frac{\partial u_2}{\partial y} + v_1\frac{\partial u_1}{\partial y} + u_2\frac{\partial u_o}{\partial y}\right) + \\ M_1(u_o + pu_1 + p^2u_2 +) - Gr(\Theta_o + p\Theta_1 + p^2\Theta_2 +) \\ -Gc(\Phi_o + p\Phi_1 + p^2\Phi_2 +) \end{bmatrix}$$

$$\frac{\partial^2 \Theta_o}{\partial y^2} + p\frac{\partial^2 \Theta_1}{\partial y^2} + p^2\frac{\partial^2 \Theta_2}{\partial y^2} + p^3\frac{\partial^2 \Theta_3}{\partial y^2} + .. = p\begin{bmatrix} u_o\frac{\partial \Theta_o}{\partial x} + p\left(u_o\frac{\partial \Theta_1}{\partial x} + u_1\frac{\partial \Theta_o}{\partial x}\right) + p^2\left(u_o\frac{\partial \Theta_2}{\partial x} + u_1\frac{\partial \Theta_1}{\partial x} + u_2\frac{\partial \Theta_o}{\partial x} + ..\right) \\ v_o\frac{\partial \Theta_o}{\partial y} + p\left(v_o\frac{\partial \Theta_1}{\partial y} + v_1\frac{\partial \Theta_o}{\partial y}\right) + p^2\left(v_o\frac{\partial \Theta_2}{\partial y} + v_1\frac{\partial \Theta_1}{\partial y} + v_2\frac{\partial \Theta_o}{\partial y}\right) - \\ N\,Pr(\Theta_o + p\Theta_1 + p^2\Theta_2 +) + Ec\begin{bmatrix} \left(\frac{\partial u_o}{\partial y}\right)^2 + p\left[\left(\frac{\partial u_o}{\partial y}\right)\left(\frac{\partial u_1}{\partial y}\right) + \left(\frac{\partial u_1}{\partial y}\right)\left(\frac{\partial u_o}{\partial y}\right)\right] \\ +p^2\left[\left(\frac{\partial u_o}{\partial y}\right)\left(\frac{\partial u_2}{\partial y}\right) + \left(\frac{\partial u_1}{\partial y}\right)^2\right] + \end{bmatrix} \\ Dr\left(\frac{\partial^2 \Phi_o}{\partial y^2} + p\frac{\partial^2 \Phi_1}{\partial y^2} + p^2\frac{\partial^2 \Phi_2}{\partial y^2} +\right) \end{bmatrix}$$

$$\frac{\partial^2 \Phi_o}{\partial y^2} + p\frac{\partial^2 \Phi_1}{\partial y^2} + p^2\frac{\partial^2 \Phi_2}{\partial y^2} + p^3\frac{\partial^2 \Phi_3}{\partial y^2} + .. = p\begin{bmatrix} u_o\frac{\partial \Phi_o}{\partial x} + p\left(u_o\frac{\partial \Phi_1}{\partial x} + u_1\frac{\partial \Phi_o}{\partial x}\right) + p^2\left(u_o\frac{\partial \Phi_2}{\partial x} + u_1\frac{\partial \Phi_1}{\partial x} + u_2\frac{\partial \Phi_o}{\partial x} + ..\right) \\ v_o\frac{\partial \Phi_o}{\partial y} + p\left(v_o\frac{\partial \Phi_1}{\partial y} + v_1\frac{\partial \Phi_o}{\partial y}\right) + p^2\left(v_o\frac{\partial \Phi_2}{\partial y} + v_1\frac{\partial \Phi_1}{\partial y} + v_2\frac{\partial \Phi_o}{\partial y}\right) + \\ Sc\delta(\Phi_o + p\Phi_1 + p^2\Phi_2 +) + Sr\left(\frac{\partial^2 \Theta_o}{\partial y^2} + p\frac{\partial^2 \Theta_1}{\partial y^2} + p^2\frac{\partial^2 \Theta_2}{\partial y^2} +\right) \end{bmatrix}$$

Equating the coefficients of the powers of p gives

$$p^o: \frac{\partial^2 u_o}{\partial y^2} = 0 \tag{21}$$

$$\frac{\partial^2 \Theta_o}{\partial y^2} = 0 \tag{22}$$

$$\frac{\partial^2 \Phi_o}{\partial y^2} = 0 \tag{23}$$

$$p: \frac{\partial^2 u_1}{\partial y^2} = u_o \frac{\partial u_o}{\partial x} + v_o \frac{\partial u_o}{\partial y} \_ M_1 u_o - Gr\Theta_o - Gc\Phi_o \tag{24}$$

$$\frac{\partial^2 \Theta_1}{\partial y^2} = u_o \frac{\partial \Theta_o}{\partial x} + v_o \frac{\partial \Theta_o}{\partial y} - N\,Pr\,\Theta_o + Ec\left(\frac{\partial u_o}{\partial y}\right)^2 + Dr\frac{\partial^2 \Phi_o}{\partial y^2} \tag{25}$$

$$\frac{\partial^2 \Phi_1}{\partial y^2} = u_o \frac{\partial \Phi_o}{\partial x} + v_o \frac{\partial \Phi_o}{\partial y} + Sc\delta\Phi_o + Sr\frac{\partial^2 \Theta_o}{\partial y^2} \tag{26}$$

$$p^2: \frac{\partial^2 u_2}{\partial y^2} = \left(v_o \frac{\partial u_1}{\partial y} + v_1 \frac{\partial u_o}{\partial y}\right) + \left(u_o \frac{\partial u_1}{\partial x} + u_1 \frac{\partial u_o}{\partial x}\right) + M_1 u_1 - Gr\Theta_1 - Gc\Phi_1 \tag{27}$$

$$\frac{\partial^2 \Theta_2}{\partial y^2} = \left(u_o \frac{\partial \Theta_1}{\partial x} + u_1 \frac{\partial \Theta_o}{\partial x}\right) + \left(v_o \frac{\partial \Theta_1}{\partial y} + v_1 \frac{\partial \Theta_o}{\partial y}\right) - N\,Pr\,\Theta_1 + Ec\left[\left(\frac{\partial u_o}{\partial y}\right)\left(\frac{\partial u_1}{\partial y}\right) + \left(\frac{\partial u_1}{\partial y}\right)\left(\frac{\partial u_o}{\partial y}\right)\right] + Dr\frac{\partial^2 \Phi_1}{\partial y^2} \tag{28}$$

$$\frac{\partial^2 \Phi_2}{\partial y^2} = \left(u_o \frac{\partial \Phi_1}{\partial x} + u_1 \frac{\partial \Phi_o}{\partial x}\right) + \left(v_o \frac{\partial \Phi_1}{\partial y} + v_1 \frac{\partial \Phi_o}{\partial y}\right) + Sc\delta\Phi_1 + Sr\frac{\partial^2 \Theta_1}{\partial y^2} \tag{29}$$

$$p^3: \frac{\partial^2 u_3}{\partial y^2} = \left(u_o \frac{\partial u_2}{\partial x} + u_1 \frac{\partial u_1}{\partial x} + u_2 \frac{\partial u_o}{\partial x} + ..\right) + \left(v_o \frac{\partial u_2}{\partial y} + v_1 \frac{\partial u_1}{\partial y} + u_2 \frac{\partial u_o}{\partial y}\right) + M_1 u_2 - Gr\Theta_2 - Gc\Phi_2 \tag{30}$$

$$\frac{\partial^2 \Theta_3}{\partial y^2} = \left(u_o \frac{\partial \Theta_2}{\partial x} + u_1 \frac{\partial \Theta_1}{\partial x} + u_2 \frac{\partial \Theta_o}{\partial x} + ..\right) + \left(v_o \frac{\partial \Theta_2}{\partial y} + v_1 \frac{\partial \Theta_1}{\partial y} + u_2 \frac{\partial \Theta_o}{\partial y}\right) - N\,Pr\,\Theta_2$$
$$+ Ec\left[\left(\frac{\partial u_o}{\partial y}\right)\left(\frac{\partial u_2}{\partial y}\right) + \left(\frac{\partial u_1}{\partial y}\right)^2\right] + Dr\frac{\partial^2 \Phi_2}{\partial y^2} \tag{31}$$

$$\frac{\partial^2 \Phi_3}{\partial y^2} = \left(u_o \frac{\partial \Phi_2}{\partial x} + u_1 \frac{\partial \Phi_1}{\partial x} + u_2 \frac{\partial \Phi_o}{\partial x}\right) + \left(v_o \frac{\partial \Phi_2}{\partial y} + v_1 \frac{\partial \Phi_1}{\partial y} + v_2 \frac{\partial \Phi_o}{\partial y}\right) + Sc\delta\Phi_2 + Sr\frac{\partial^2 \Theta_2}{\partial y^2} \tag{32}$$

It is noteworthy that in HPM, the boundary conditions $y = 0$serve as the solutions to the zeroth-order problems i.e.

$$u = u_o(x,0) = 0, v = v_o(x,0) = 0, v_1(x,0) = 0, v_2(x,0) = 0, \Theta_o = exp(ikx), \Phi_o = exp(ikx) \tag{33}$$

such that $u_o(x,0)$, $\Theta_o(x,0)$, and $\Phi_o(x,0)$ are not a function of y but x only.

However, in the Modified Homotopy Perturbation Method, the zeroth-order problems are solved using the boundary conditions $u_o(x,0)$, $\Theta_o(x,0)$, and $\Phi_o(x,0)$ are not a function of y but x only (see Momani *et al.*) [48]. In this paper, equations (21) – (45) are solved using the Homotopy Perturbation Method. By this, the first, second, and third-order equations reduce to

$$p: \frac{\partial^2 u_1}{\partial y^2} = -Gr\Theta_o - Gc\Phi_o \tag{34}$$

$$\frac{\partial^2 \Theta_1}{\partial y^2} = -N\,Pr\,\Theta_o + Dr\frac{\partial^2 \Phi_o}{\partial y^2} \tag{35}$$

$$\frac{\partial^2 \Phi_1}{\partial y^2} = Sc\delta\Phi_o + Sr\frac{\partial^2 \Theta_o}{\partial y^2} \tag{36}$$

with the boundary conditions

$$u_1(x,0) = 0, u_1(x,\infty) = 0, \Theta_1(x,0) = 0, \Theta_1(x,\infty) = 0, \Phi_1(x,0) = 0, \Phi_1(x,\infty) = 0 \tag{37}$$

The solution obtained are functions of x and y such that

$u_1(x,y)$, $\Theta_1(x,y)$, and $\Phi_1(x,y)$

$$p^2: \frac{\partial^2 u_2}{\partial y^2} = M_1 u_1 - Gr\Theta_1 - Gc\Phi_1 \tag{38}$$

$$\frac{\partial^2 \Theta_2}{\partial y^2} = u_1\frac{\partial \Theta_o}{\partial x} - N\,Pr\,\Theta_1 + Dr\frac{\partial^2 \Phi_1}{\partial y^2} \tag{39}$$

$$\frac{\partial^2 \Phi_2}{\partial y^2} = u_1\frac{\partial \Phi_o}{\partial x} + Sc\delta\Phi_1 + Sr\frac{\partial^2 \Theta_2}{\partial y^2} \tag{40}$$

with the boundary conditions

$$u_2(x,0) = 0, u_2(x,\infty) = 0, \Theta_2(x,0) = 0, \Theta_2(x,\infty) = 0, \Phi_2(x,0) = 0, \Phi_2(x,\infty) = 0 \tag{41}$$

Additionally, the solution obtained are functions of x and y.

$$p^3: \frac{\partial^2 u_3}{\partial y^2} = u_1\frac{\partial u_1}{\partial x} + M_1 u_2 - Gr\Theta_2 - Gc\Phi_2 \tag{42}$$

$$\frac{\partial^2 \Theta_3}{\partial y^2} = \left(u_1\frac{\partial \Theta_1}{\partial x} + u_2\frac{\partial \Theta_o}{\partial x}\right) + N\,Pr\,\Theta_2 + Ec\left(\frac{\partial u_1}{\partial y}\right)^2 + Dr\frac{\partial^2 \Phi_2}{\partial y^2} \tag{43}$$

$$\frac{\partial^2 \Phi_3}{\partial y^2} = \left(u_1\frac{\partial \Phi_1}{\partial x} + u_2\frac{\partial \Phi_o}{\partial x}\right) + Sc\delta\Phi_2 + Sr\frac{\partial^2 \Theta_2}{\partial y^2} \tag{44}$$

with the boundary conditions

$$u_3(x,0) = 0, u_3(x,\infty) = 0, \Theta_3(x,0) = 0, \Theta_3(x,\infty) = 0, \Phi_3(x,0) = 0, \Phi_3(x,\infty) = 0 \tag{45}$$

Similarly, the solution obtained are functions of x and y

Other factors affecting the flow system are the Nusselt number, Sherwood number, and Wall Shear Stress prescribed dimensionless as

$$Nu = -\Theta'(y)|_{y=0} \tag{46}$$

$$Sh = -\Phi'(y)|_{y=0} \tag{47}$$

$$Cf = f''(y)|_{y=0} \tag{48}$$

Results and Discussion

The effects of wave number and magnetic field on the boundary layer flow over an accelerating vertical surface in the presence of heat generation, reacting species, and zero-time periodic boundaries are examined. For physically realistic constant values of $\chi = 0.1, Gc = 1.0, Gc = 1.0, \chi = 0.3, \delta = 0.5, Pr = 0.7, Sc = 0.62, N = 0.5$ and varied values of k, M and Dr, we obtained the shown in Table 1 - Table 11, we obtained the figures and tables below.

Table 1: Effects of Wave Number on the Concentration

Φ	$k=0.5$	$k=1.0$	$k=1.5$	$k=2.0$	$k=2.5$
0.0	2.12755	19.5689	68.4191	164.151	322.382
0.5	2.15936	20.5866	84.67797	288.67330	925.42120
1.0	2.55605	30.4161	770.813	6301.62	30912.16
1.5	4.43593	516.663	9814.62	75820.27	366227.69
2.0	16.2891	2673.33	49450.2	380141.4	1.834*10^6
2.5	55.7371	9401.068	172884.3	1.328*10^6	6.402*10^6

Table 2: Effects of Wave Number on the Temperature

Θ	$k=0.5$	$k=1.0$	$k=1.5$	$k=2.0$	$k=2.5$
0.0	2.12755	19.5690	68.4191	164.151	322.382
0.5	2.09154	20.0065	83.4328	290.946	951.702
1.0	1.63484	25.4370	616.411	4980.24	24352.17
1.5	1.76067	299.056	5543.21	42564.5	205140.3
2.0	10.9395	1186.30	18710.3	134499.5	627142.5
2.5	74.2473	6275.20	78148.5	461602.0	1.84*10^6

Table 3: Effects of Wave Number on the Velocity

u	$k=0.5$	$k=1.0$	$k=1.5$	$k=2.0$	$k=2.5$
0.0	0.00737	0.00157	0.00380	0.01002	0.02511
0.5	0.19861	1.85326	6.47382	15.5349	30.5273
1.0	2.99825	27.6443	96.6431	231.871	455.410
1.5	9.38819	86.4479	302.235	725.127	1424.15
2.0	21.2644	195.713	684.254	1641.67	3224.19
2.5	41.4344	381.263	1332.99	3198.11	6280.97

Table 4: Effects of Wave Number on the Sherwood number, Nusselt number, and Skin Friction

k	Sh	Nu	Cf
0.5	0.06814	0.07660	0.41756
1.0	2.96106	2.06302	3.82808
1.5	46.8636	45.1759	13.3858
2.0	349.540	358.226	32.1143
2.5	1671.54	1742.85	63.0654

The effects of the Wave number on the flow are seen in Table 1 – Table 4. They show that the increase in the Wave number increases the concentration, temperature, velocity, Sherwood number, Nusselt number, and the stress on the wall. While the frequency of a propagating wave is the number of waves per unit of time, the Wave number is the number of waves or cycles per unit distance and is represented $k = 1/\lambda = 2\pi/\lambda$ (metre^{-1}). The higher the number of waves or cycles in a given distance, the higher the strength of the disturbance/agitation of the fluid particles. And, the increase in the particle agitation produces a resultant increase in the concentration, temperature, and velocity. More so, the higher the fluid concentration, temperature, and velocity the higher the Sherwood number, Nusselt number, and force on the wall, respectively.

Table 5: Effects of Magnetic Field on the Concentration

Φ	$M = 0.5$	$M = 1.0$	$M = 1.5$	$M = 2.0$	$M = 2.5$
0.5	1.01226	1.01229	1.01233	1.01236	1.01239
1.0	1.20378	1.20384	1.20390	1.20396	1.20402
1.5	1.59074	1.59358	1.59643	1.59929	1.60216
2.0	2.31960	2.35139	2.38361	2.41625	2.44930
2.5	4.01517	4.20536	4.39790	4.59251	4.78894

Table 6: Effects of Magnetic Field on the Temperature

Θ	$M = 0.5$	$M = 1.0$	$M = 1.5$	$M = 2.0$	$M = 2.5$
0.5	0.98049	0.98050	0.98052	0.98054	0.98057
1.0	0.77405	0.77696	0.78147	0.78759	0.79532
1.5	0.55621	0.81801	1.24537	1.83777	2.59279
2.0	2.07593	6.59853	13.6545	23.2409	35.3571
2.5	15.60191	50.08857	103.841	176.859	269.143

Table 7: Effects of Magnetic Field on the Velocity

u	$M = 0.5$	$M = 1.0$	$M = 1.5$	$M = 2.0$	$M = 2.5$
0.5	70.9581	111.410	151.862	232.766	232.766
1.0	120.509	189.489	258.471	396.435	396.435
1.5	169.360	266.683	364.012	558.674	558.674
2.0	217.191	342.378	467.582	718.002	718.002
2.5	263.583	415.768	567.979	872.426	872.426

Furthermore, the effects of the Magnetic field on the flow are shown in Table 5 – Table 8 depict that the increase in the magnetic field strength increases the concentration, temperature, and velocity in the range of$M < 1.5$. In particular, the velocity remains constant for $M \geq 2.0$. The presence of reacting species in a fluid, its particles exist as ions/charges. Similarly, the presence of magnetic field influence in the flow system produces electric currents. The magnetic field interaction with the electric currents produces a mechanical force, the Lorentz force (a drag force) that has the potency of reducing the flow velocity. Velocity is a function of concentration and temperature/energy. They must decrease with velocity Therefore, the increase in the concentration, temperature, and velocity (in the range of $M < 1.5$) as the magnetic field strength increases must be due to some other factors.

Table 8: Effects of Dufour Number on the Concentration

Φ	$Dr = 0.05$	$Dr = 0.1$	$Dr = 0.3$	$Dr = 0.5$	$Dr = 0.9$
0.05	1.01229	1.01229	1.01229	1.01230	1.01230
0.1	1.20376	1.20377	1.20378	1.20386	1.20390
0.3	1.59069	1.59071	1.59074	1.59097	1.59109
0.5	2.31950	2.31955	2.31960	2.32004	2.32025
0.9	4.01503	4.01510	4.01517	4.01580	4.01611

Table 9: Effects of Dufour Number on the Temperature

Θ	$Dr = 0.05$	$Dr = 0.1$	$Dr = 0.3$	$Dr = 0.5$	$Dr = 0.9$
0.05	0.98042	0.98049	0.98079	0.98108	0.98168
0.1	0.77305	0.77405	0.77804	0.78204	0.79003
0.3	0.55363	0.55621	0.56653	0.57685	0.59751
0.5	2.07258	2.07593	2.08938	2.10292	2.13025
0.9	15.59907	15.60191	15.61329	15.62473	15.64777

Table 10: Effects of Dufour Number on the Velocity

u	$Dr = 0.05$	$Dr = 0.1$	$Dr = 0.3$	$Dr = 0.5$	$Dr = 0.9$
0.05	60.33535	70.95808	113.4496	155.9415	240.9256
0.1	102.3903	120.5091	192.9897	265.4736	410.4443
0.3	143.753	169.3603	271.8139	374.2827	579.2333
0.5	184.1154	217.1906	349.5742	482.0083	746.9209
0.9	223.0915	263.5833	425.7902	588.1425	912.9728

More so, the effects of Dufour number on the flow are seen in Table 8 – Table 10. They depict that an increase in the Dufour number increases the concentration, temperature, and velocity. In highly interacting flow systems where magnetic field, convection, and reacting species are present, heat and mass transfer occur simultaneously. Cross-diffusion effects have a significant influence on fluid buoyancy. Dufour (thermo-diffusion) and Soret (Diffusion-Thermo/Thermo-phoresis) are generated by concentration and temperature differentials, respectively. Usually, for smallness, cross-diffusion effects are taken as second-order phenomena whose size and effects are smaller than Fourier and Fick's law. Specifically, the buoyancy force loses the fluid particles from the grip of the fluid viscosity, thus making them more mobile, interactive, and energetic. As these occur, the fluid velocity increases, and in turn, the concentration and energy/temperature increase.

Conclusion

We considered the effects of Wave number and magnetic field on the boundary layer flow over an accelerating vertical surface in the presence of heat generation, reacting species, and zero-time periodic boundaries. The analysis of results shows that an increase in

- Wave number increases the concentration, temperature, velocity, Sherwood, Nusselt number, and stress on the wall.
- The Magnetic field increases the concentration, temperature, and velocity in the range of $0 < M < 1.5$.
- Dufour number increases the concentration, temperature, and velocity.

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