International Journal of Physics and Applications

E-ISSN: 2664-7583
P-ISSN: 2664-7575
IJOS 2024; 6(2): 16-24

www.physicsjournal.in
Received: 06-06-2024
Accepted: 12-07-2024

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CMBR Constrained Milky-Way Mass
Resolving The Hubble Tension

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DOI: https://doi.org/10.33545/26647575.2024.v6.i2a.93

Abstract
Total Milky-Way (MW) Mass (M_G) is a longstanding question, without clear & precise resolution. An array of estimates exist, many of which seem to partially agree, partially disagree or totally disagree with each other. To address the question of MW Mass, we test numerous estimates for compliance with the Power Spectrum Hubble Constant (H_0), & the Cosmic Microwave Background Radiation (CMBR) Temperature (T_0). It is shown that intermediate MW Mass estimates (*e.g.* $1.54 \bullet 10^{12}$ $M_\odot$) breach Particle Data Group (PDG) requirements for the satisfaction of H_0 & T_0 constraints, whilst solutions less than $7 \bullet 10^{11}$ $M_\odot$ are substantially more compliant. Hence, the Ideal Solution for Virial Mass ($M_G = M_{Vir}$) & Virial Radius (r_{Vir}) precisely satisfying H_0 & T_0, are $6.3768 \bullet 10^{11}$ $M_\odot$ & 182.6 (kpc) respectively. Moreover, the Ideal M_{Vir} result was found to be quasi-identical to the 2008 value of $6.3142 \bullet 10^{11}$ $M_\odot$ derived by *Storti*, demonstrating that the Solar Distance to Galactic Centre (R_0) requires Astro Physical Community (APC) standardisation. In addition, we propose that M82 denotes a potential candidate Galaxy which may be utilised to invalidate the existence of the 'Hubble Tension'. Furthermore, we also demonstrate that the existence of the 'Hubble Tension' can be invalidated via the experimental observation of a sufficiently large population of Galaxies.

Keywords: CMBR, Hubble Constant, Hubble Tension, Milky-Way, Virial Mass, Virial Radius, Virial Velocity

1 Introduction
1.1 Nomenclature

Table 1: Nomenclature

Symbol	Description
BPT	Buckingham-Π-Theory
$H_\odot$	Astronomical Hubble Constant (*Flat Universe*); also termed the Distance Ladder Hubble Constant: See Eq. (1): $H_\odot = H(r_{Circ}, M_{Circ})$
H_0	Cosmological Hubble Constant (*Flat Universe*); also termed the Power Spectrum Hubble Constant: See Eq. (1): $H_0 = H(R_0, M_{Vir})$ & $H_0 = H(R_\odot, M_{Vir})$
M_G	Total Milky-Way Mass
M_Φ	Total Cosmological Mass
M_{Circ}	Encircled Mass: The Total Matter estimated (*or measured*) to be encircled within an assigned (*or measured*) radius defined by 'r_{Circ}'
M_{Vir}	Virial Mass: The Gravitationally Bound Total Matter within a Galactic Structure
r_{Vir}	Virial Radius: The Radial Limit of the Gravitationally Bound Total Matter within a Galactic Structure: See Eq. (2)
PDG	Particle Data Group
R_0, $R_\odot$	Solar Distance to Galactic Centre: R_0 = 8.122 (kpc) = Gravity Collaboration (GC: 2018) R_0 = 8.178 (kpc) = Particle Data Group (PDG: 2019) $R_\odot$ = 8.29 (kpc) = McMillan (2011) R_0 = 8.34 (kpc) = Huang (2016)
R_Φ	Hubble Radius (*Length*)
T_0	Cosmic Microwave Background Radiation (CMBR) Temperature: See Eq. (4)
V_{Vir}	Virial Velocity: Denotes the Velocity of the Gravitationally Bound Total Matter within a Galactic Structure at the Virial Radius See Eq. (3)

1.2 Overview
This article presents a novel approach to solving the M_G problem. We achieve this via the application of BPT [1], which produces *exact solutions* as shown by Eq. (1, 4-7). To assist readers, we have published a MS-Excel **S**preadsheet **C**alculator (SC), supported by a Math Cad **S**olution **A**lgorithm (SA), which details how the SC is constructed. The SA presents a method for determining M_G, when constrained by H_0 & T_0. Hence, M_{Vir} & r_{Vir} are scaled-up to M_Φ & R_Φ whilst being constrained by T_0. Thus, we obtain a system of equations which solve for M_{Vir} & r_{Vir} simultaneously; we shall address the principle of $M_G = M_{Vir}$ in a subsequent section.

1.3 How does BPT solve the M_G problem?
If we consider a Galactic Sphere of Virial Radius whilst assuming an absence of significant matter at this great distance (*e.g.* > 182 kpc, *i.e.* > 595 kLyr), & homogenous CMBR measurements, then the Galactic Sphere will be in thermal equilibrium with a Cosmological Sphere of Hubble Radius at T_0 = 2.7255 (K). Thus, by preserving Dynamic, Kinematic & Geometric Similarity between the Galactic Sphere & the Cosmological Sphere, we may apply BPT to constrain both spheres simultaneously to H_0 & T_0. Once constrained, we are able to accurately predict M_G explicitly utilising the Solar Distance to Galactic Centre, instead of $\mathbf{r_{Vir}}$.

1.4 What does the application of BPT require in this instance?
In the context of M_G & the manner in which we are utilising physical scaling techniques to solve the problem via Galactic & Cosmological Spheres, we require the AstroPhysical Community (APC) to agree upon a single value of R_0. Although present differences in R_0 estimates may seem innocuous to many people, this is not the case. Our results demonstrate that the consequences of diverse R_0 opinion, has significant impact on the value of M_G; for example, the computed difference in M_G, between utilising R_0 = 8.122 (kpc) or R_0 = 8.34 (kpc), is approximately 13 (%). Thus, for consistency & reliability, we have standardised upon R_0 = 8.178 (kpc) as the authoritative reference throughout this article; we shall revisit the issue of authoritative references in a subsequent section.

1.5 Does the Hubble Tension cause an issue?
As discussed in the Overview, our MathCad SA presents a method for determining M_G, when constrained by H_0 & T_0. Hence, an obvious question arises; *i.e.* does the Hubble Tension cause an issue with this approach?
1. No; we explicitly demonstrate that the Hubble Tension is fictitious.
2. The Hubble Tension is fictitious because we utilise Eq. (1) to compute experimentally verified solutions to both forms of Hubble Constant [2]; *i.e.* Cosmological & Astronomical Hubble Constants (H_0, $H_\odot$) respectively.
3. The fact that Eq. (1) produces both Hubble Constants, simply by supplying the appropriate inputs, validates our assertion regarding Hubble Tension.
4. Hence, the Hubble Constant is not actually constant, but varies spatially & temporally; ergo, regions of Space-Time expand at different rates depending upon their M_{Vir} content.
5. The T_0 constraint is calculated from the H_0 constraint utilising Eq. (4), therefore it is unaffected by the Hubble Tension.

1.6 How does our approach differ from convention?
The method we develop for constraining M_G to H_0 & T_0, is an extension of the conventional technique of Virial Envelope Projection (VEP). The conventional approach involves defining a radius encircling a known or estimated content of mass; r_{Circ} & M_{Circ} respectively. From this region, M_{Vir} & r_{Vir} estimates are projected. Since r_{Vir} is a non-physical boundary, the specific assumptions utilised by researchers has resulted in a significant diversity of outcomes. The step-by-step procedure for VEP is clearly defined in the published SC & graphically summarised by Fig. (1). The key advancement this article achieves beyond the conventional technique, is the concept of thermal equilibrium at T_0 = 2.7255 (K) between a Galactic Sphere of Virial Radius, & a Cosmological Sphere of Hubble Radius. This facilitates the employment of a Standard Engineering Scaling Technique (termed BPT), in order to constrain M_{Vir} & r_{Vir} to H_0 & T_0.

1.7 Key Point Summary
The following significant results are achieved:
1. We derive precise solutions for M_{Vir}, r_{Vir} & V_{Vir}, constrained by H_0 & T_0.
2. We confirm that the Hubble Tension is fictitious & does not exist; *i.e.* both forms of Hubble Constant are determined utilising Eq. (1); hence, no so-called 'tension' exists between H_0 & $H_\odot$.
3. We show that some of the estimates developed by Eadie, & all of the estimates developed by Watkins, Karukes & Jiao may be discarded.

1.8 Associated Viewing
We recommend all readers to review the following video content; it relates explicitly, or in large part, to this article & will assist comprehension.
- BPT: https://youtu.be/kQzbrr8rsVc?si=e9kNXbZBWS1XnhGy
- Deriving the Present-Epoch Values of H_0 & $H_\odot$; *Resolving the Hubble Tension*: https://youtu.be/79wutrnJUWQ?si=cTb7j4-A8EREFFtn
- Deriving the Present-Epoch Value of T_0: https://youtu.be/8j2nO_Nc2NI?si=2Ppea3E3eXP0IdjB
- Deriving the Present-Epoch Value of M_G: https://youtu.be/v5YLFUQgmpU?si=10ViCRBBiJJDUELQ
- The History of The Cosmos; *From The Big-Bang to The Present-Epoch*: https://youtu.be/kKj4CrFX6Lo?si=HMWuO-i_xVbW5x9Y
- Deriving the Cosmological History of Λ, Ω_Λ, Ω_M & q: https://youtu.be/ljFIxxWgOsU?si=mHdQ5PJmLuX_5iSj
- Deriving the Cosmological History of H_0 & T_0: https://youtu.be/hN_hoXgLNlQ?si=qUCw1iJgd0nKk_b9

2 Method
2.1 Dilemma
How much does the Milky-Way, actually weigh? This is a longstanding question, without clear & precise resolution. An array of estimates exist, many of which seem to partially agree, partially disagree or totally disagree with each other [1]. To make matters alarmingly worse, since the publication of the 2016 Bland-Hawthorn & Gerhard Annual Review in Astronomy & Astrophysics, a stream of discordant predictions for M_G have appeared in the scientific literature. In fact, one author has significantly increased their estimate over a period of four years. In part, one of the causes of this

discordance, is the diversity in the concept of M_G associated with research objectives. For example, some authors explicitly refer to Virial Mass in their outcomes, whilst other authors imply Dynamical Mass but do not explicitly state it. Moreover, a generalised claim of M_G is often utilised, but without clear delineation with respect to Viral or Dynamical Masses. Hence, it has become very clear that the ongoing search for M_G requires some degree of standardisation. In this article, we propose that the most appropriate metric to be discussed & quantified in this field of research, is M_{Vir}. However, before we move on to further discussion of M_{Vir}, we shall also emphasise other areas where discordance exists. This will assist in comprehending the magnitude of the 'discordance problem' in contemporary Astronomy & Astrophysics.

To the interested mind seeking an answer to discordance, the landscape of opinion is somewhat overwhelming & confusing. Reasons for this appear to be the diversity of underlying assumptions between solutions, the diversity of authoritative parameter usage, & the diversity of measurement tolerances. To bring these points into sharper focus, [2] define the Solar Distance to Galactic Centre as $R_\odot = 8.29$ (kpc) ± 0.16 (kpc) [*i.e.* ± 2 (%) error], whilst the PDG Value is defined as $R_0 = 8.178$ (kpc) ± 0.013 (kpc) [3] [*i.e.* ± 0.16 (%) error]. Both of these estimates originate from authoritative sources, yet their central values differ [≈ 1.4(%)] & their limiting values barely overlap [$R_\odot = 8.13$ (kpc) vs. $R_0 = 8.191$ (kpc)]. The issue with this conflict is that follow-up researchers utilising this parameter within their M_G constructs, introduce a sprinkle of 'faith' into their solutions via their choice of parameter standard.

Herein, we standardise on the PDG value of R_0 due to the tighter associated tolerance [± 0.013 (kpc)]. Similarly & for consistency, we standardise on PDG values when available; including their value of Cosmological Hubble Constant $H_0 = 67.4$ (km/s/Mpc) ± 0.5 (km/s/Mpc) [*i.e.* ± 0.74 (%) error]. Although we have standardised on PDG information as our source of truth, by necessity we are required to seek alternative sources whenever the PDG lacks the information we require. Consequently, we recognise [4] as our standard for the Astronomical Hubble Constant $H_\odot = 72.8$ (km/s/Mpc) ± 1.6 (km/s/Mpc) [*i.e.* ± 2.2 (%) error]. Although the associated error is greater than that of H_0, we are left with little choice as the diversity of opinion within this domain is often conflicted [1]. To temper our concerns, we have selected a reference value for $H_\odot$ in precise agreement with the central value stated in [5] [*i.e.* 72.8 (km/s/Mpc)]. Hence, we recognise the following sources as yielding authoritative limits, which are utilised in our Spreadsheet Calculator (SC):

- PDG CMBR [3]:
  $T_0 = 2.7255$ (K) ± ΔT_0; $\Delta T_0 = 0.0006$ (K)
- PDG Solar Mass [3]: $M_\odot = 1.98841 \bullet 10^{30}$ (kg)
- Solar Distance to Galactic Centre:
  McMillan (2011) [2]: $R_\odot = 8.29$ (kpc) ± $\Delta R_\odot$; $\Delta R_\odot = 0.16$ (kpc)
  PDG [3]: $R_0 = 8.178$ (kpc) ± ΔR_0; $\Delta R_0 = 0.013$ (kpc)
- PDG Cosmological Hubble Constant: H_0 [3]:
  $H_0 = 67.4$ (km/s/Mpc) ± ΔH_0; $\Delta H_0 = 0.5$ (km/s/Mpc)
- Dhawan (2018) Astronomical Hubble Constant: $H_\odot$ [4]:
  $H_\odot = 72.8$ (km/s/Mpc) ± $\Delta H_\odot$; $\Delta H_\odot = 1.6$ (km/s/Mpc)
- Liao (2020) Non-Flat Universe Astronomical Hubble Constant: $H_{\odot\odot}$ [5]:
  $H_{\odot\odot} = 77.3$ (km/s/Mpc),
  [$+\Delta H_{\odot\odot} = 2.2$, $-\Delta H_{\odot\odot} = 3$] (km/s/Mpc)

2.2 Milky-Way Virial Mass & Trending Estimates

The characteristics [3] of M_{Vir} may be concisely summarised as follows

- M_{Vir} relates the Total Kinetic Energy (TKE) of a self-gravitating body, due to the motions of its constituent parts, to the Gravitational Potential Energy (GPE) of the self-gravitating body.
- The Virial Theorem (VT) forms the root of many galaxy scaling relations.
- The comparison of mass estimates based upon the VT, to estimates based upon the luminosities of galaxies, is one technique used by Astronomers to detect the presence of Dark Matter in galaxies and clusters of galaxies.

Hence, we may interpret M_G estimates appearing in key scientific literature, as the gravitationally bound M_{Vir}; according to the following set of discordant estimates:

- Eadie (2017) [6]: $M_G = M_{Vir} = 6.2 \bullet 10^{11}$ $M_\odot$
- Watkins (2019) [7]: $M_G = M_{Vir} = 1.54 \bullet 10^{12}$ $M_\odot$
- Eadie (2019) [8]: $M_G = M_{Vir} = 7 \bullet 10^{11}$ $M_\odot$
- Karukes (2020) [9]: $M_G = M_{Vir} = 8.9 \bullet 10^{11}$ $M_\odot$
- Eadie (2021) [10]: $M_G = M_{Vir} = 1.08 \bullet 10^{12}$ $M_\odot$
- Jiao (2023) [11]: $M_G = M_{Vir} = 2.06 \bullet 10^{11}$ $M_\odot$

Where, the average value for the set is given by $M_G = M_{Vir} = 8.39 \bullet 10^{11}$ $M_\odot$. Thus, three obvious questions arise:

- How can one author (*i.e.* Eadie) have significantly increased their mass estimate over a period of four (4) years (2017-2021)?
- Why is the Jiao estimate, so far below its peer contemporaries?
- Does the average value of $M_G = M_{Vir}$, denote reasonable middle-ground?

To answer these questions simultaneously, we shall consider a more consistent set of estimates, derived by *Storti* as follows:

- Conference Proceedings (2008) [12]:
  $M_G = M_{Vir} = 6.3142 \bullet 10^{11}$ $M_\odot$
- This article (2023): $M_G = M_{Vir} = 6.3768 \bullet 10^{11}$ $M_\odot$

So then, why has the *Storti* set of estimates remained so consistent over so many years? The answer to this question is that the *Storti* estimates are constrained by H_0 & T_0 [13]. In this article, we present a mathematical solution for M_{Vir} utilising Eq. (1), which is then constrained to the CMBR by Eq. (4); hence, the three (3) questions we have identified regarding the discordant set of estimates - vanish.

2.3 Relieving the Hubble Tension

At this juncture, it is important to explain our introduction of terminology differentiating between H_0 & $H_\odot$; the Cosmological & Astronomical Hubble Constants respectively. The purpose of this differentiation is to reflect the so-called 'tension' between H_0 & $H_\odot$. Our position is that no such tension exists & both results are simultaneously correct, thus both results may be generated from a single exact solution & indeed, the Standard Model of Cosmology (SMoC) would be in peril if $H_0 = H_\odot$. We relieve the 'tension' by utilising an exact solution common to both forms of Hubble Constant for the Present-Epoch given by Eq. (1) formulated utilising BPT, as follows [13];

$$H(r,M) = K_G \cdot \frac{m_{\gamma\gamma}}{h} \cdot \ln\left[\frac{\sqrt{(3\cdot\pi)^{7\cdot\mu} \cdot 32^{\mu^2}}}{256} \cdot \ln\left[\frac{(3\cdot\pi)^{\mu}}{4} \cdot \left(\frac{m_h}{M}\right)^{\mu^2} \cdot \left(\frac{r}{\lambda_h}\right)^{7\cdot\mu^2}\right]^{7\cdot\mu} \cdot \left(\frac{m_h}{M}\right)^{5\cdot\mu^2} \cdot \left(\frac{r}{\lambda_h}\right)^{26\cdot\mu^2}\right]^5 \quad (1)$$

Where

K_G is termed the Experimental Relationship Function (ERF): This denotes a required attribute of the BPT method, which relates mathematical prediction to experimental observation. Since Eq. (1) was formulated specifically for use with MW Data (*i.e.* R_0), its value with respect to Eq. (1) is $K_G = 1$; signifying a perfect relationship between our derivation & experimental observation. However, if another Galaxy were to be selected & the derivation process repeated, then K_G may take other values (*e.g.* ½, ⅓, ⅔, 2, 3, 4 etc.).

$m_{\gamma\gamma}$ = Photon Mass-Energy = $3.19515507344683 \cdot 10^{-45}$ (eV)[14]:

Important Considerations

1. Massless photons are an assumption; they have not been experimentally measured to be massless.
2. The PDG provisions for massive photons [15].
3. The two extremities in nature which have not been experimentally verified are absolute zero ‘anything’, or infinity ‘anything’; nor can they ever be experimentally verified. Insistence upon the existence of these extremities, is an issue of faith, not science.

- h = Planck’s Constant = $6.62607015 \cdot 10^{-34}$ (J/Hz)
- m_h = Planck Mass = $5.45551186133462 \cdot 10^{-8}$ (kg)
- λ_h = Planck Length = $4.05135054323488 \cdot 10^{-35}$ (m)
- μ = Mathematical Constant = 1/3

Such that

$H_0 = H(r, M)$ = Cosmological Hubble Constant

Where

$r = R_0$ [$\pm \Delta R_0$], or as required, $r = R_\odot$ [$\pm \Delta R_\odot$]

$M = M_{Vir}$ = Virial Mass

$G = 6.6743 \cdot 10^{-11}$ ($m^3/kg/s^2$)

r_{Vir} = Virial Radius

$$r_{Vir} = \sqrt[3]{\frac{G \cdot M}{100 \cdot H(r, M)^2}} \quad (2)$$

V_{Vir} = Virial Velocity

$$V_{Vir} = \sqrt{\frac{G \cdot M}{r_{Vir}}} \quad (3)$$

$H_\odot = H(r, M)$ = Astronomical Hubble Constant

Where

$r = r_{Circ}$ = Encircling Radius

$M = M_{Circ}$ = Encircled Mass

Note

- Both forms of Hubble Constant are determined utilising Eq. (1); hence, no so-called ‘tension’ exists between H_0 & $H_\odot$.
- Since r_{Vir} is significantly greater than R_0, the Inter-Galactic Space-Time Manifold at r_{Vir} may be usefully approximated as being flat. Therefore, the temperature at r_{Vir} is equal to the CMBR.

How Eq. (1) works

1. Eq. (1) utilises ($r = R_0$) & ($M = M_{Vir}$) to compute H_0; subsequently, enabling the computation of r_{Vir} & V_{Vir}.
2. Similarly, Eq. (1) utilises ($r = r_{Circ}$) & ($M = M_{Circ}$) to compute $H_\odot$.
3. Eq. (1) is derived from the application of a widely utilised & commonly applied Engineering technique, responsible for formulating many important Thermodynamic & Fluid Mechanical numbers; such that Dynamic, Kinematic & Geometric Similarity is preserved between two physical systems utilising BPT. Hence, M_{Vir} & r_{Vir} are scaled-up to M_Φ & R_Φ, whilst being constrained by T_0. In essence, one obtains a system of equations which are solved simultaneously; consequently, key cosmological properties [Ω_Λ, H_Φ] were predicted & experimentally verified in [13, 16].

2.4 Constraining M_{Vir} Estimates Utilising T_0

Utilising T_0 as formulated by *Storti* [13, 16], in accordance with the *exact solutions* given by Eq. (4-7), we are able to tightly constrain M_{Vir} estimates as follows:

$$T_0(H) = K_W \cdot St_T \cdot \ln\left(\frac{H_\alpha}{H}\right) \cdot H^{5 \cdot \mu^2} \quad (4)$$

$$St_T = \frac{4 \cdot \mu}{(4 \cdot \mu)^\mu \cdot c} \cdot \left(\frac{\lambda_x \cdot \omega_h^2}{\pi}\right)^{2 \cdot \mu^2} \quad (5)$$

$$H_\alpha = \frac{\omega_h}{\lambda_x} \quad (6)$$

$$\lambda_x = 4 \cdot \sqrt{\frac{2 \cdot \mu}{\pi^\mu}} \quad (7)$$

Where

ω_h = Planck Frequency = $7.3998153159224 \cdot 10^{42}$ (Hz)

$H_\Box$ (*Big-Bang Hubble Constant*): pg. 12, Eq. (24) [17]

H_0 (*which contains $M_G = M_{Vir}$*) is obtained utilising Eq. (1)

St_T^9 (*Physical Constant*): pg. 14, Eq. (42) [17]

K_W = Wien Displacement Constant = 2.897771955 (mm•K)

λ_x = Mathematical Constant; derivation: https://youtu.be/TJWJAFu82xw?si=RPMbJuqzKsDbDtnY

μ = Mathematical Constant = 1/3

Note

The complete April-2007 derivation of Eq. (1, 4-6) is available in [17].

The complete April-2007 derivation of Eq. (7) is available in [18].

Experimental validation of Eq. (1, 4-7) is summarised in Tab. (2) below

Table 2: Historical Validation of Eq. (1, 4-7)

PDG (2008) [19] (Measurement)	Storti (2008) [12] Storti (2009) [20] (Prediction)	PDG (2019) [3] (Measurement)
H_0 = 73 (km/s/Mpc)	H_0 = 67.0843 (km/s/Mpc)	H_0 = 67.4 (km/s/Mpc)
R_0 = 8.0 (kpc)	R_0 = 8.1072 (kpc)	R_0 = 8.178 (kpc)

$T_0 = 2.725$ (K)	$T_0 = 2.7248$ (K)	$T_0 = 2.7255$ (K)
$M_G = M_{Vir}$ = N/A	$M_G = M_{Vir} = 6.3142 \bullet 10^{11}\ M_\odot$	$M_G = M_{Vir}$ = N/A

3. Results

3.1 Overview

The results presented in this article, denote a subset of data extracted from a significantly larger set. Our complete archive of results are derived, analysed & described within the following artefacts:

1. Milky-Way Mass (Data Analysis) [21].
2. Milky-Way Mass (Spreadsheet Calculator) [22].
3. Milky-Way Mass (Solution Algorithm) [23].
4. Milky-Way Mass (Spreadsheet Calc. User Tutorial): https://youtu.be/ovLbvj3HuNM?si=lw9r27LC8rRgUjf4

3.2 The Significance of the Solar Distance to Galactic Centre

By considering a Galactic Sphere of Virial Radius (r_{Vir}), as existing in thermal equilibrium with a Cosmological Sphere of Hubble Radius (R_Φ) at $T_0 = 2.7255$ (K), we may apply BPT to constrain both spheres to H_0 & T_0, simultaneously. Once constrained, we are able to accurately predict M_G explicitly utilising the Solar Distance to Galactic Centre, instead of r_{Vir}**.** Thus, by preserving Dynamic, Kinematic & Geometric Similarity between the Galactic & Cosmological Spheres, our application of BPT facilitates simultaneous solutions of H_0, T_0, M_{Vir}, r_{Vir} & V_{Vir} as follows:

Note

It is necessary to express our computed results to many significant figures, in order to communicate the thermal difference between solutions.

Solution-1 utilises the Gravity Collaboration (GC-2018) value of R_0:

- Input: $R_0 = 8.122$ (kpc) [24]
- Outputs: pg. 32-33 [23]:
- $H_0 = 67.1181448809845$ (km/s/Mpc)
- $T_0 = 2.72550000185347$ (K)
- $M_G = M_{Vir} = 6.15255690713876 \bullet 10^{11}\ M_\odot$
- $r_{Vir} = 180.431470785913$ (kpc)
- $V_{Vir} = 121.102255972981$ (km/s)

Solution-2 utilises the PDG-2019 value of R_0

- Input: $R_0 = 8.178$ (kpc) [3]
- Outputs: pg. 30 [23]:
- $H_0 = 67.1181447833258$ (km/s/Mpc)
- $T_0 = 2.72549999967897$ (K)
- $M_G = M_{Vir} = 6.37679587662515 \bullet 10^{11}\ M_\odot$
- $r_{Vir} = 182.597396787976$ (kpc)
- $V_{Vir} = 122.555985146738$ (km/s)

Solution-3 utilises the McMillan (2011) value of $R_\odot$

- Input: $R_\odot = 8.29$ (kpc) [2]
- Outputs: pg. 33 [23]:
- $H_0 = 67.1181447812731$ (km/s/Mpc)
- $T_0 = 2.72549999963327$ (K)
- $M_G = M_{Vir} = 6.84509179041111 \bullet 10^{11}\ M_\odot$
- $r_{Vir} = 186.962067505131$ (kpc)
- $V_{Vir} = 125.485471154156$ (km/s)

Solution-4 utilises the Huang (2016) value of R_0

- Input: $R_0 = 8.34$ (kpc) [25]
- Outputs: pg. 32-33 [23]:
- $H_0 = 67.1181447977433$ (km/s/Mpc)
- $T_0 = 2.7255$ (K)
- $M_G = M_{Vir} = 7.06292958098438 \bullet 10^{11}\ M_\odot$
- $r_{Vir} = 188.924682900692$ (kpc)
- $V_{Vir} = 126.802742227964$ (km/s)

Thus, the computed difference in M_G, between utilising $R_0 = 8.122$ (kpc), or $R_0 = 8.34$ (kpc), is approximately 13 (%).

3.3 Testing Key Scientific Literature

Test conditions

- We apply the PDG value of $R_0 = 8.178$ (kpc) [3], for Eadie, Karukes & Jiao testing. However, Watkins explicitly states the use of $R_\odot = 8.29$ (kpc) in their research article; hence, we test the Watkins estimate utilising $R_\odot$ [7].
- PDG H_0 limits must be satisfied.
  $H_0 = 66.9$ to 67.9 (km/s/Mpc) [3].
- PDG T_0 limits must be satisfied.
  $T_0 = 2.7249$ to 2.7261 (K) [3].

As identified previously, key scientific literature generates discordant M_G estimates. Thus, testing these estimates against our BPT construct, as appears on pg. 41 [23], yields the following Tab. (3) test results:

Table 3: ($M_G = M_{Vir}$) Estimate Test Results

		Test Results			
M_G Estimate		**H_0 (km/s/Mpc)**	**T_0 (K)**	**r_{Vir} (kpc)**	**V_{Vir} (km/s)**
Eadie (2017) [6]	$6.2 \bullet 10^{11}\ M_\odot$	67.1	2.7259	180.9	121.4
Watkins (2019) [7]	$1.54 \bullet 10^{12}\ M_\odot$	66.6	2.7146	246.2	164
Eadie (2019) [8]	$7 \bullet 10^{11}\ M_\odot$	67.1	2.7242	188.5	126.4
Karukes (2020) [9]	$8.9 \bullet 10^{11}\ M_\odot$	66.9	2.721	204.5	136.8
Eadie (2021) [10]	$1.08 \bullet 10^{12}\ M_\odot$	66.8	2.7184	218.3	145.9
Jiao (2023) [11]	$2.06 \bullet 10^{11}\ M_\odot$	67.8	2.7408	124.4	84.4
Average Mass (M_{AV})	$8.39 \bullet 10^{11}\ M_\odot$	67	2.7218	200.4	134.2

Thus, it becomes apparent that the only M_G estimate which satisfies PDG H_0 & T_0 limits [3], is Eadie (2017) [6]; as appears in Tab. (4):

Table 4: ($M_G = M_{Vir}$) Estimate Compliance Matrix

M_G Estimate	**H_0 Compliance**	**T_0 Compliance**
Eadie (2017) [6]	Yes	Yes
Watkins (2019) [7]	No	No

Eadie (2019) [8]	Yes	No
Karukes (2020) [9]	Yes	No
Eadie (2021) [10]	No	No
Jiao (2023) [11]	Yes	No
M_{AV}	Yes	No

3.4 MW Ideal Astro Statistical Solution Framework

As revealed by Tab. (4), the *only* M_G estimate satisfying PDG H_0 & T_0 constraints, is Eadie (2017) [6]. Given that all Eadie M_G estimates from 2015 to 2019 have been generated utilising AstroStatistical methods [26, 27, 6, 8], we may develop a Milky-Way (MW) Ideal AstroStatistical Solution Framework constrained by PDG H_0 & T_0 limits [3], represented by Fig. (1) & Tab. (5, 6), as follows:

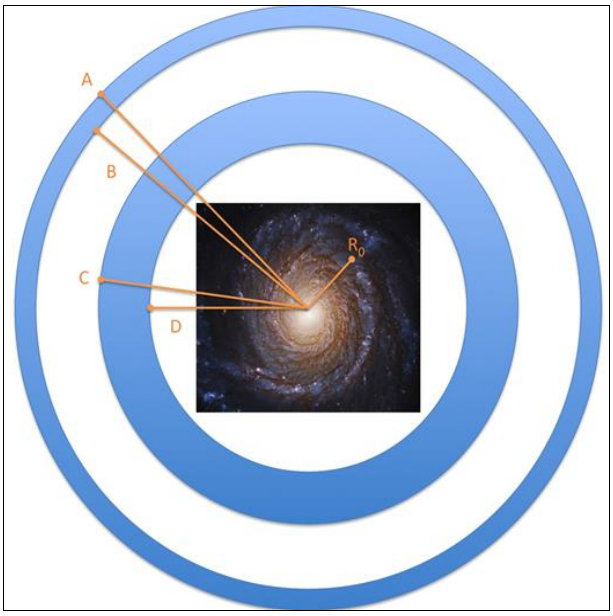

Fig 1: MW Ideal AstroStatistical Solution Framework: pg. 28 [23]

Table 5: MW Ideal AstroStatistical Solution Framework (*Region: C-D*)

Parameter	Value at Virial Limit (A)	Value at Virial Limit (B)
M_{Vir}	$6.67 \bullet 10^{11}$ $M_\odot$	$6.1 \bullet 10^{11}$ $M_\odot$
r_{Vir}	185.4 (kpc)	179.9 (kpc)
V_{Vir}	124.4 (km/s)	120.8 (km/s)
H_0	67.09 (km/s/Mpc)	67.15 (km/s/Mpc)
T_0	2.7249 (K)	2.7261 (K)

Table 6: MW Ideal AstroStatistical Solution Framework (*Region: A-B*)

Parameter	Value at Encircling Limit (C)	Value at Encircling Limit (D)
M_{Circ}	$7.4 \bullet 10^{11}$ $M_\odot$	$2 \bullet 10^{11}$ $M_\odot$
r_{Circ}	76.7 (kpc)	23.1 (kpc)
$H_\odot$	74.4 (km/s/Mpc)	71.2 (km/s/Mpc)

Note

- The M_{Vir}, r_{Vir} & V_{Vir} values shown on Tab. (5), at the Virial Limits (A) & (B), are constrained by H_0 & T_0 limits.
- The r_{Circ} values shown on Tab. (6), at the Encircling Limits (C) & (D), are constrained by M_{Circ} & $H_\odot$ limits.
- Celestial objects observed within region (C-D), will tend to imply $H_\odot$ values within the range indicated by Tab. (6).
- Fig. (1) & Tab. (5, 6) explains the origin of the so-called Hubble Tension. An analogous regional distribution surrounds all large scale galactic structures. Thus, the location where a celestial object resides within its host galaxy, will effect the value of $H_\odot$ observed; thereby biasing Cosmic Distance Ladder Measurements & implying the existence of the Hubble Tension.
- The Hubble Constant is not actually constant, but varies spatially & temporally; ergo, regions of Space-Time expand at different rates depending upon their M_{Vir} content.

The MW Ideal AstroStatistical Solution at T_0 = 2.7255 (K) is calculated as follows:

1. H_0 = 67.12 (km/s/Mpc) [13]: pg. 30 [23]
2. $M_{Vir} = 6.3768 \bullet 10^{11}$ $M_\odot$
3. r_{Vir} = 182.6 (kpc)
4. V_{Vir} = 122.6 (km/s)

[3.5] Other Galaxies

By utilising Eq. (1) in Astronomical Form (*i.e.* $H_\odot$) when K_G = 1, we may constrain the Apparent Mass (M_{App}) & Apparent Radius (r_{App}) of various Galaxies to T_0, as described by Eq. (4), in order to compute their associated Virial Properties in accordance with Tab. (7) as follows: pg. (42-45) [23]

Table 7: Galactic Virial Properties Constrained by CMBR (T_0)

Galaxy	Apparent Estimates		Virial Results Constrained by T_0		
	$M_{App} = M_{Circ}$ ($\bullet 10^9$ $M_\odot$)	$r_{App} = r_{Circ}$ (kLyr)	$M_G = M_{Vir}$ ($\bullet 10^9$ $M_\odot$)	r_{Vir} (kpc)	V_{Vir} (km/s)
M31	1,230	110	1,384	236.4	158.7
M32	3	3.25	2	26.4	17.7
M33	50	30	54	80.3	53.9
M51	160	30	169	117.3	78.8
M63	110	49	122	105.2	70.6
M65	250	45	273	137.6	92.4
M82	50	18.5	51	78.7	52.8
M100	200	53.5	221	128.3	86.1
M101	1,000	85	1,116	220	147.7
M104	800	25	783	195.5	131.2

Where, the *guesstimates* for M_{App} & r_{App} have been taken from various internet resources, as appears in Tab. (8) as follows:

Table 8: Galactic Information Sources

Galaxy	Information Source
M31	https://g.co/kgs/wEhCyb
M32	https://g.co/kgs/e13asY
M33	https://g.co/kgs/ufyt9t
M51	https://g.co/kgs/KeW1ty
M63	https://g.co/kgs/MMJvk2
M65	https://g.co/kgs/NAqsA2
M82	https://g.co/kgs/PXeWVf
M100	https://astropixels.com/galaxies/M100-01.html
M101	https://g.co/kgs/xYJMmw
M104	https://science.nasa.gov/mission/Hubble/science/explore-the-night-sky/Hubble-messier-catalog/messier-104/

Hence, by inserting ($M_G = M_{Vir}$) & ($r_{App} = r_{Circ}$) as appears in Tab. (7), into Eq. (1) when K_G = 1, we may estimate the expected value of $H_\odot$ to be observed at the major axis periphery of the various Galaxies appearing in Tab. (9): pg. (44) [23]

Table 9: Est. Astronomical Hubble Constant ($H_\odot$) Measurement

Galaxy	$M_G = M_{Vir}$ ($\bullet\ 10^9\ M_\odot$)	$r_{App} = r_{Circ}$ (kLyr)	$H_\odot$ (km/s/Mpc)	H_0 Tolerance Compliant
M31	1,384	110	71.2	No
M32	2	3.25	64	No
M33	54	30	69	No
M51	169	30	68.3	No
M63	122	49	70.1	No
M65	273	45	69.3	No
M82	51	18.5	67.5	Yes
M100	221	53.5	70	No
M101	1,116	85	70.5	No
M104	783	25	66.8	No
Average Value of $H_\odot$			68.67	Yes
Standard Deviation of $H_\odot$			2.13	

Where, the tolerance for H_0 is defined as follows:
PDG Cosmological Hubble Constant: H_0 [3]:
$H_0 = 67.4$ (km/s/Mpc) $\pm \Delta H_0$; $\Delta H_0 = 0.5$ (km/s/Mpc)

Thus, *assuming* that the values of M_{App} & r_{App} defined by the internet resources appearing in Tab. (8) are accurate, then M82 is the only Galaxy appearing in Tab. (9) which is capable of producing an observed vale of $H_\odot$ within the H_0 range specified by the PDG:

- Experimental confirmation of this prediction requires the existence of a suitable candidate celestial object at the major axis periphery of M82.
- If such an observation is confirmed, then the existence of the 'Hubble Tension' has been invalidated; *i.e.* the Astronomical Hubble Constant as determined via Distance Ladder Measurements ($H_\odot$), will fall within the range of the Cosmological Hubble Constant as determined by Power Spectrum Measurements (H_0).
- The average value of $H_\odot$ appearing in Tab. (9) is 68.67 (km/s/Mpc), with a Standard Deviation of 2.13 (km/s/Mpc); implying that the average value of $H_\odot$ tends to H_0 as the number of experimentally confirmed Galactic Observations tends to infinity. Hence, once again, resolving the 'Hubble Tension' problem.

4. Discussion

The results presented demonstrate a relationship between M_{Vir}, r_{Vir}, H_0 & T_0. We have developed a technique to determine the mass & size of the MW Galaxy, which exists in thermal equilibrium with the CMBR. Since r_{Vir} is many times greater than R_0, we have developed a standardised mechanism by which to quantify M_G; *i.e.* when $M_G = M_{Vir}$. Thus, by adopting our methodology as an industry standard, the discordance of estimates for M_G we presently experience in the scientific literature - vanish.

The need to adopt an agreed standard of estimation, constrained by the CMBR, is emphasised by the dramatic change in M_{Vir} we observe, occurring with Eadie from 2017 to 2021. We have shown that Eadie estimated M_G most accurately in 2017, but has dramatically altered position in 2021. Standardising upon our methodology would help to prevent these major contradictions from reoccurring. The primary reason our methodology may be considered reliable is because *Storti* predicted the value of H_0, five (5) years in advance of experimental confirmation of Eq. (1, 4-7) in 2008 [12], by the Planck Satellite in 2013 [28]. It is important to appreciate the sequence of historical events:

1. [2008] Storti: $H_0 = 67.0843$ (km/s/Mpc) [12]
2. [2008] PDG: $H_0 = 73$ (km/s/Mpc) [29]
3. [2013] PDG: $H_0 = 67.3$ (km/s/Mpc) [28]
4. [2019] PDG: $H_0 = 67.4$ (km/s/Mpc) [3]
5. [2020] Storti: $H_0 = 67.1181$ (km/s/Mpc) [13]

Thus, *Storti's* 2008 prediction which debunked scientific dogma in 2013, is ample evidence that Eq. (1, 4-7) should be adopted as an industry standard; experimentally verified prediction is a core feature of the scientific method. Consequently, our results demonstrate that all $M_G = M_{Vir}$ estimates not explicitly constrained by H_0 & T_0, may be discarded. Therefore, we have determined that the mass & size of the MW may be described according to:

- $M_{Vir} = 6.3768 \bullet 10^{11}\ M_\odot$
- $r_{Vir} = 182.6$ (kpc)
- $V_{Vir} = 122.6$ (km/s)

It should also be communicated to readers that, prior to selecting Eadie (2017) [6], Watkins (2019) [7], Eadie (2019) [8], Karukes (2020) [9], Eadie (2021) [10] & Jiao (2023) [11] for scrutiny herein, we rigorously tested many of the citations quoted by these authors. The complete list of additionally tested M_G estimates appears as follows:
Klypin (2002) [30], Bovy (2015) [31], Price-Whelan (2017) [32], Sohn (2018) [33], Vasiliev (2019) [34], Kafle (2012) [35], Küpper (2015) [36], Posti (2019) [37], Wilkinson (1999) [38], Sakamoto (2003) [39], Deason (2012) [40], Gibbons (2014) [41], Eadie (2020) [42], Erkal (2019) [43], Carlesi (2022) [44]

5. Conclusion

In this article, we have tested numerous Milky-Way (MW) Mass (M_G) estimates for compliance with the Power Spectrum Hubble Constant (H_0), & the Cosmic Microwave Background Radiation (CMBR) Temperature (T_0). It was shown that intermediate MW Mass estimates (*e.g.* $1.54 \bullet 10^{12}\ M_\odot$) breach Particle Data Group (PDG) requirements for the satisfaction of H_0 & T_0 constraints, whilst solutions less than $7 \bullet 10^{11}\ M_\odot$ are substantially more compliant. Hence, the Ideal Solution for Virial Mass ($M_G = M_{Vir}$) & Virial Radius (r_{Vir}) precisely satisfying H_0 & T_0, are $6.3768 \bullet 10^{11}\ M_\odot$ & 182.6 (kpc) respectively. Moreover, the Ideal M_{Vir} result was found to be quasi-identical to the 2008 value of $6.3142 \bullet 10^{11}\ M_\odot$ derived by *Storti*, demonstrating that the Solar Distance to Galactic Centre (R_0) requires AstroPhysical Community (APC) standardisation. In addition, *assuming* that the values of M_{App} & r_{App} defined by the internet resources appearing in Tab. (8) are accurate, then M82 denotes a potential candidate Galaxy which may be utilised to invalidate the existence of the 'Hubble Tension' as appears in Tab. (9). Furthermore, Tab. (9) implies that, the existence of the 'Hubble Tension' can be invalidated via the experimental observation of a sufficiently large population of Galaxies such that the average value of $H_\odot$ tends to H_0.

6. Data Availability

The data & analysis associated with the citations listed herein, may be accessed via the following artefacts:

- Milky-Way Mass (Data Analysis) [21]
- Milky-Way Mass (Spreadsheet Calculator) [22]
- Milky-Way Mass (Solution Algorithm) [23]
- Milky-Way Mass (Spreadsheet Calc. User Tutorial): https://youtu.be/ovLbvj3HuNM?si=lw9r27LC8rRgUjf4
- The complete April-2007 derivation of Eq. (1, 4-6) is available in [17]
- The complete April-2007 derivation of Eq. (7) is available in [18]

- Experimental validation of Eq. (1, 4-7) is summarised in Tab. (2)

7. Funding

No funding is associated with the generation of this research article.

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9. Appendix

At the time of formulating this research article, the PDG Value of R_0 had remained constant from 2019-2022. However, in August-2023, the PDG Value of R_0 transitioned from 8.178 (kpc) to 8.275 (kpc) [45] . Consequently, this 1.2 (%) increase in the value of R_0, modifies our H_0 & T_0 constrained values of M_{Vir}, r_{Vir} & V_{Vir}, according to:

- $M_{Vir} = 6.7808 \bullet 10^{11} M_\odot$: Estimated Increase = 6.3 (%)
- r_{Vir} = 186.4 (kpc): Estimated Increase = 2.1 (%)
- V_{Vir} = 125.1 (km/s): Estimated Increase = 2.1 (%)

Similarly, Tab. (1) Transforms to Tab. (10) as follows:

Parameter	Value at Virial Limit (A)	Value at Virial Limit (B)
M_{Vir}	$7.13 \bullet 10^{11} M_\odot$	$6.45 \bullet 10^{11} M_\odot$
r_{Vir}	189.6 (kpc)	183.2 (kpc)
V_{Vir}	127.2 (km/s)	123 (km/s)
H_0	67.09 (km/s/Mpc)	67.15 (km/s/Mpc)
T_0	2.7249 (K)	2.7261 (K)

Tab. (10): MW Ideal AstroStatistical Solution Framework (Region: A-B): Refer to Fig. (1)

Moreover, for the first time in PDG history (August-2023), a distinction is drawn between H_0 & $H_\odot$; formally recognising the existence of the so-called 'Hubble Tension'. Consequently, the PDG-2023 Value of Astronomical Hubble Constant [4] is given as $H_\odot = 73$ (km/s/Mpc) $\pm \Delta H_\odot$; $\Delta H_\odot = 1$ (km/s/Mpc). Hence, Tab. (6) transforms to Tab. (11) as follows:

Parameter	Value at Encircling Limit (C)	Value at Encircling Limit (D)
M_{Circ}	$7.4 \bullet 10^{11} M_\odot$	$2 \bullet 10^{11} M_\odot$
r_{Circ}	68.2 (kpc)	29.4 (kpc)
$H_\odot$	74 (km/s/Mpc)	72 (km/s/Mpc)

Tab. (11): MW Ideal AstroStatistical Solution Framework (Region: C-D): Refer to Fig. (1)

Note

The PDG-2023 changes & additions defined above, do not constitute a significant deviation from the construct presented in preceding sections.