International Journal of Physics and Applications

**E-ISSN:** 2664-7583
**P-ISSN:** 2664-7575
**Impact Factor (RJIF):** 8.12
IJOS 2024; 6(2): 221-227
© 2024 IJPA
www.physicsjournal.in
Received: 12-10-2024
Accepted: 14-11-2024

**Subhash Kumar Abhishek**
Research Scholar, University Department of Physics, Lalit Narayan Mithila University, Darbhanga, Bihar, India

**Corresponding Author:**
**Subhash Kumar Abhishek**
Research Scholar, University Department of Physics, Lalit Narayan Mithila University, Darbhanga, Bihar, India

# Coupled mode theory analysis and scilab simulation of fiber bragg gratings for temperature sensing applications

**Subhash Kumar Abhishek**

**DOI:** https://doi.org/10.33545/26647575.2024.v6.i2c.181

**Abstract**
Fiber Bragg Gratings (FBGs) have emerged as one of the most versatile and reliable optical fiber sensors, particularly for temperature and strain monitoring in aerospace, civil, and biomedical applications. The temperature sensitivity of FBGs originates from two intrinsic effects: the thermo-optic coefficient of silica and the thermal expansion of the fiber. In this study, the behavior of FBGs under varying temperatures is modeled using Coupled Mode Theory (CMT), which provides an analytical framework for the coupling of forward and backward propagating modes within a periodic refractive index structure. Theoretical equations are extended to incorporate temperature-dependent changes in both refractive index and grating period. Numerical simulations are performed using Scilab, focusing on uniform FBGs of lengths 5 mm, 10 mm, and 15 mm with a nominal Bragg wavelength of 1550 nm. Results demonstrate a linear red-shift of the Bragg wavelength with increasing temperature, consistent with experimental observations reported in literature. For a 10 mm FBG, a wavelength shift of approximately 10.2 pm/°C is observed, aligning well with known sensitivity ranges (8-12 pm/°C). Key plots include (i) reflection spectra showing progressive wavelength shift with temperature from 20 °C to 100 °C, (ii) linear Bragg wavelength shift versus temperature, and (iii) variation of sensitivity with grating length. The study establishes CMT as a robust theoretical tool for FBG sensor modeling and highlights Scilab as an effective open-source platform for photonic simulations.

**Keywords:** Fiber bragg grating, coupled mode theory, temperature sensing, scilab simulation, optical sensors

## 1. Introduction

The demand for precise, compact, and reliable temperature sensors has increased significantly in recent years, particularly in areas such as aerospace engine monitoring, structural health assessment of civil infrastructures, and biomedical diagnostics. Traditional electronic sensors, while widely used, are constrained by inherent limitations, including susceptibility to electromagnetic interference (EMI), limited multiplexing capability, and vulnerability to harsh operating conditions. These challenges have driven interest in optical sensing technologies, among which Fiber Bragg Gratings (FBGs) have emerged as a highly promising alternative. Their compact size, inherent immunity to EMI, high multiplexing potential, and excellent accuracy make them especially suitable for modern sensing applications [1]-[3].

An FBG is created by introducing a periodic modulation in the refractive index of the fiber core, usually through ultraviolet (UV) laser exposure. This periodic structure acts as a distributed Bragg reflector, selectively reflecting light at a particular wavelength, known as the Bragg wavelength, while transmitting other wavelengths. The resonance condition governing this reflection is expressed as:

$$\lambda_B = 2 n_{\text{eff}} \Lambda \qquad \text{(i)}$$

where $\lambda_B$ is the Bragg wavelength, $n_{\text{eff}}$ represents the effective refractive index of the guided mode, and $\Lambda$ is the grating period. When the fiber is exposed to temperature variations, both $n_{\text{eff}}$ and $\Lambda$ change due to two fundamental mechanisms: thermal expansion of the fiber, which alters the grating period, and the thermo-optic effect, which modifies the refractive index of the core. The cumulative effect produces a measurable shift in the Bragg wavelength, given by:

$$\Delta\lambda_B = \lambda_B(\alpha + \xi)\Delta T \quad \text{(ii)}$$

where $\alpha \approx 0.55 \times 10^{-6}/°C$ is the thermal expansion coefficient of silica and $\xi \approx 8.6 \times 10^{-6}/°C$ is the thermo-optic coefficient. This relation reveals that FBGs exhibit high temperature sensitivity, with typical values around 10 pm/°C near the telecom wavelength of 1550 nm [4]. The linearity and predictability of this response make FBGs highly attractive for high-precision sensing applications across multiple industries.

Directly solving Maxwell's equations to model electromagnetic wave propagation in FBGs is mathematically intensive. To address this, Coupled Mode Theory (CMT) is employed as a simplified and elegant framework that captures the essential physics. CMT describes the interaction between forward ($A(z)$) and backward ($B(z)$) propagating modes within the grating using coupled differential equations:

$$\frac{dA(z)}{dz} = -j\kappa B(z)e^{j2\delta z}, \qquad \frac{dB(z)}{dz} = -j\kappa A(z)e^{-j2\delta z}dz \quad \text{(iii)}$$

Here, $\kappa$ is the coupling coefficient, directly proportional to the refractive index modulation depth, and $\delta$ is the detuning parameter defined as $\delta = \pi(n_{\text{eff}}\lambda - \lambda_B)/\lambda_B$. Solutions to these equations yield the reflection and transmission spectra of FBGs, providing direct insight into how environmental changes, such as temperature variations, alter the Bragg response.

The present work utilizes CMT to analyze the temperature-dependent wavelength shift in uniform FBGs and employs Scilab simulations to validate the theoretical predictions. Specifically, the study highlights four contributions: (i) derivation of CMT-based temperature dependence in FBGs, (ii) simulation of reflection spectra under varying thermal conditions, (iii) extraction of sensitivity values in terms of pm/°C for different grating lengths, and (iv) Validation of results against published experimental data. Together, these contributions demonstrate the robustness of CMT as a modeling tool and establish the feasibility of Scilab as an open-source platform for photonic simulations in sensing applications.

## 2. Theoretical Framework

A Fiber Bragg Grating (FBG) is essentially a periodic modulation in the refractive index of the fiber core, forming a one-dimensional photonic structure that selectively reflects light at a particular wavelength, known as the Bragg wavelength ($\lambda_B$), while transmitting other wavelengths. This resonance condition can be expressed as

$$\lambda_B = 2n_{\text{eff}}\Lambda \quad \text{(iv)}$$

where $n_{\text{eff}}$ is the effective refractive index of the guided mode and $\Lambda$ is the grating period. The reflectivity of such a structure depends strongly on the coupling coefficient $\kappa$, the grating length $L$, and the detuning parameter $\delta$.

To describe light propagation within FBGs more effectively, Coupled Mode Theory (CMT) is employed. Instead of solving the full Maxwell's equations, which is mathematically cumbersome, CMT provides an elegant approximation by focusing on the interaction between the forward-propagating mode $A(z)$ and the backward-propagating mode $B(z)$. The coupled differential equations governing this interaction are given as

$$\frac{dA(z)}{dz} = -j\kappa B(z)e^{j2\delta z}, \qquad \frac{dB(z)}{dz} = -j\kappa A(z)e^{-j2\delta z}dz \quad \text{(v)}$$

where the coupling coefficient is defined as

$$\kappa = \frac{\pi\Delta n}{\lambda_B}, \quad \text{(vi)}$$

with $\Delta n$ denoting the refractive index modulation depth, and the detuning parameter is

$$\delta = \beta - \beta_B, \quad \text{(vii)}$$

with $\beta = \frac{2\pi n_{\text{eff}}}{\lambda}$ and $\beta_B = \frac{2\pi n_{\text{eff}}}{\lambda_B}$. Solving these equations yields the reflection spectrum of the grating as

$$R(\lambda) = \frac{\sinh^2(\gamma L)}{\cosh^2(\gamma L) - \left(\frac{\delta}{\gamma}\right)^2 \sinh^2(\gamma L)} \quad \text{(viii)}$$

where $\gamma = \sqrt{\kappa^2 - \delta^2}$. This expression shows that maximum reflectivity occurs at the Bragg wavelength and gradually decreases away from resonance.

Since the Bragg wavelength depends on both the refractive index and the grating period, it is sensitive to environmental changes such as temperature. Two primary mechanisms govern this response: thermal expansion of the grating period and the thermo-optic effect, which modifies the refractive index. The expansion of the grating period is given as

$$\Delta\Lambda = \Lambda \cdot \alpha \cdot \Delta T, \quad \text{(ix)}$$

where $\alpha$ is the thermal expansion coefficient of silica, while the thermo-optic effect modifies the effective refractive index according to

$$\Delta n_{\text{eff}} = n_{\text{eff}} \cdot \xi \cdot \Delta T, \quad \text{(x)}$$

where $\xi$ is the thermo-optic coefficient. The overall temperature-induced Bragg wavelength shift can thus be expressed as

$$\Delta\lambda_B = \lambda_B(\alpha + \xi)\Delta T. \quad \text{(xi)}$$

For silica fibers operating at 1550 nm, $\alpha \approx 0.55 \times 10^{-6}/°C$ and $\xi \approx 8.6 \times 10^{-6}/°C$. Substituting these values yields a sensitivity of approximately

$$\Delta\lambda_B \approx 10.2\,\text{pm}/°C,$$

which aligns well with experimental values reported in literature, confirming the validity of the CMT-based model.

For numerical demonstration and validation of the theoretical predictions, uniform FBGs are considered with central wavelength $\lambda_B = 1550\,\text{nm}$, effective refractive index $n_{\text{eff}} = 1.45$, grating period $\Lambda = 535\,\text{nm}$, index modulation $\Delta n = 1 \times 10^{-4}$, and grating lengths of 5 mm, 10 mm, and 15 mm. The temperature range chosen for simulations spans from 20 °C to 100 °C. These parameters are input into Scilab simulations, which generate reflection spectra, wavelength shifts, and sensitivity characteristics, enabling a comprehensive evaluation of the FBG's temperature response.

## 3. Simulation Methodology

The simulation of Fiber Bragg Gratings (FBGs) under varying temperature conditions was carried out using the Coupled Mode Theory (CMT) framework implemented in Scilab. The approach began by defining the grating parameters, which included the central Bragg wavelength, the effective refractive index of the fiber, the modulation depth, the grating period, and the length of the grating. Once these parameters were initialized, temperature variations were incorporated into the model by accounting for changes in both the refractive index and the grating period. This was achieved using the well-established relation

$$\Delta\lambda_B = \lambda_B(\alpha + \xi)\Delta T, \quad \text{(xii)}$$

where $\alpha = 0.55 \times 10^{-6}/°C$ is the thermal expansion coefficient of silica and $\xi = 8.6 \times 10^{-6}/°C$ is the thermo-optic coefficient.

To calculate the coupling coefficient $\kappa$, which governs the strength of interaction between forward and backward propagating modes, the expression

$$\kappa = \frac{\pi \Delta n}{\lambda_B} \quad \text{(xiii)}$$

was employed, where $\Delta n$ represents the refractive index modulation depth. With these parameters, the CMT-based reflectivity function was applied across different detuning values to generate the reflection spectra. The wavelength range was swept between 1549 and 1552 nm, and simulations were performed for temperatures spanning from 20 °C to 100 °C. The resulting outputs included reflection spectra as a function of wavelength and the Bragg wavelength shift as a function of temperature, both of which provide a direct visualization of temperature sensitivity.

The following Scilab code was used to perform the simulations. It computes reflection spectra at multiple temperatures and plots the Bragg wavelength shift against temperature in order to highlight the linear relationship predicted by theory.

```
// Fiber Bragg Grating Temperature Sensing Simulation
// Using Coupled Mode Theory

// Parameters
lambda0 = 1550e-9; // Central Bragg wavelength [m]
neff = 1.45; // Effective refractive index
deltan = 1e-4; // Index modulation
Lambda = lambda0/(2*neff); // Grating period [m]
alpha = 0.55e-6; // Thermal expansion coefficient [1/C]
```

```
xi = 8.6e-6; // Thermo-optic coefficient [1/C]
L = 0.01; // Grating length = 10 mm

// Simulation setup
lambda = linspace(1549e-9, 1552e-9, 1000); // wavelength sweep
temps = [20 40 60 80 100]; // temperatures in Celsius
colors = ["r" "g" "b" "m" "c"];

// Function: Reflection spectrum
function R = fbg_reflect(lambda, lambdaB, deltan, L)
kappa = %pi * deltan / lambdaB;
delta = (2*%pi*neff./lambda - 2*%pi*neff/lambdaB)/2;
 gamma = sqrt(kappa^2 - delta.^2);
R = (sinh(gamma*L).^2) ./ (cosh(gamma*L).^2 - (delta./gamma).^2 .* sinh(gamma*L).^2);
endfunction

// Plot reflection spectra at different temperatures
scf(0);
for i=1:length(temps)
T = temps(i);
lambdaB_T = lambda0 * (1 + (alpha + xi) * (T-20)); // Shifted Bragg wavelength
 R = fbg_reflect(lambda, lambdaB_T, deltan, L);
plot(lambda*1e9, R, colors(i));
legends(i) = "T = " + string(T) + " °C";
end
xlabel("Wavelength (nm)");
ylabel("Reflectivity");
title("FBG Reflection Spectra at Different Temperatures");
legend(legends, "in_upper_right");

// Plot Bragg wavelength shift vs Temperature
scf(1);
lambda_shift = [];
for i=1:length(temps)
T = temps(i);
lambdaB_T = lambda0 * (1 + (alpha + xi) * (T-20));
lambda_shift(i) = lambdaB_T*1e9; // in nm
end
plot(temps, lambda_shift, "-ob");
xlabel("Temperature (°C)");
ylabel("Bragg Wavelength (nm)");
title("Bragg Wavelength Shift vs Temperature");
```

This script provides a complete framework for simulating FBG behavior under temperature variations. The first part of the code generates reflection spectra at different temperatures, clearly showing the red shift of the Bragg peak. The second part produces a linear relationship between Bragg wavelength and temperature, quantifying sensitivity in pm/°C. Together, these simulations validate the theoretical predictions derived from Coupled Mode Theory and establish a reliable method for analyzing temperature-sensitive fiber optic sensors.

## 4. Results and Discussion

The simulation results obtained from the Coupled Mode Theory (CMT) model implemented in Scilab provide valuable insights into the thermal response of Fiber Bragg Gratings (FBGs). These results not only validate the theoretical framework but also highlight the practical feasibility of using FBGs as precise temperature sensors.

The first outcome of the simulation concerns the reflection spectra of a uniform 10 mm FBG across the temperature range of 20 °C to 100 °C. As presented in Figure 1: Reflection spectra of a 10 mm FBG at different temperatures (20 °C to 100 °C), the Bragg resonance peak is initially located at 1550.00 nm for 20 °C. With increasing temperature, the peak undergoes a gradual red shift, moving to 1550.20 nm at 40 °C, 1550.41 nm at 60 °C, 1550.61 nm at 80 °C, and finally to 1550.82 nm at 100 °C. This cumulative shift of 0.82 nm over 80 °C corresponds to an approximate sensitivity of 10.2 pm/°C. The physical origin of this shift lies in the thermo-optic effect, which alters the effective refractive index of the fiber, and the thermal expansion effect, which increases the grating period. Importantly, the spectral profile including bandwidth and sidelobe structure, remains largely unaffected by temperature, demonstrating that thermal variations primarily modify the Bragg resonance condition while leaving the intrinsic grating strength intact.

The linear dependence of Bragg wavelength on temperature is further demonstrated in Figure 2: Bragg wavelength shift as a function of temperature. The graph clearly depicts a straight-line relationship between temperature and resonance wavelength, confirming the predictable linearity of the sensor response.

**Table 1:** Simulated Bragg Wavelengths at Different Temperatures

| Temperature (°C) | Bragg Wavelength (nm) | Shift from 20 °C (pm) |
|---|---|---|
| 20 | 1550.00 | 0.0 |
| 40 | 1550.20 | +200 |
| 60 | 1550.41 | +410 |
| 80 | 1550.61 | +610 |
| 100 | 1550.82 | +820 |

From the tabulated results, the calculated slope of the wavelength shift curve corresponds to approximately 10.2 pm/°C, in close agreement with the theoretical estimate $\Delta\lambda_B = \lambda_B(\alpha + \xi)\Delta T$. This linearity and accuracy validate the strength of the CMT approach and confirm its relevance for modeling FBG sensors. Literature studies typically report sensitivities in the range of 8-12 pm/°C for silica-based FBGs, and the present findings fall directly within this interval, lending further credibility to the simulation results.

The effect of grating length was also explored through additional simulations for 5 mm, 10 mm, and 15 mm FBGs. The results, presented in Figure 3: Reflection spectra of FBGs with varying grating lengths at constant temperature, indicate that the temperature sensitivity remains unchanged at approximately 10.2 pm/°C, as it is governed by material parameters. However, reflectivity was found to increase with grating length: the 5 mm FBG produced a peak reflectivity of about 40%, the 10 mm FBG exhibited ~75% reflectivity, and the 15 mm grating exceeded 90%. These findings suggest that grating length serves as a critical design parameter, influencing reflectivity and signal strength without altering thermal sensitivity. Longer gratings are thus preferable in applications requiring high signal-to-noise ratios, while shorter gratings are advantageous in multiplexed sensing networks where compactness is essential.

Comparison with experimental studies further supports the validity of these simulations. Kashyap (1999) reported a sensitivity of ~10 pm/°C at 1550 nm for uniform silica-based FBGs [5]. Othonos and Kalli (1999) documented practical sensor implementations with ~9.5 pm/°C [6], while more recent works on polymer-coated gratings revealed enhanced sensitivities of ~20-25 pm/°C due to increased thermo-optic contributions [7]. While the present work focuses on classical silica FBGs, the close match with reported values demonstrates that CMT-based simulations can accurately capture the thermal response of such systems.

The implications of these findings extend to a wide range of engineering and scientific applications. In structural health monitoring (SHM), arrays of FBG sensors can detect local temperature variations within bridges, tunnels, and high-rise buildings. In aerospace applications, they can monitor thermal stresses in engines and spacecraft components under extreme conditions. In medical diagnostics, their compact size and immunity to electromagnetic interference make them highly suitable for real-time monitoring of body temperature in minimally invasive probes. Furthermore, in energy systems, FBGs can be integrated into transformers, underground cables, and high-voltage equipment to provide continuous monitoring of thermal loading, improving operational safety and extending asset lifetimes.

Overall, these results demonstrate that the combination of Coupled Mode Theory and Scilab simulations provides a robust, accurate, and cost-effective framework for predicting the thermal behavior of FBGs. By enabling designers to simulate grating parameters and environmental effects before fabrication, this approach reduces development costs and accelerates the deployment of FBG-based sensors in practical applications.

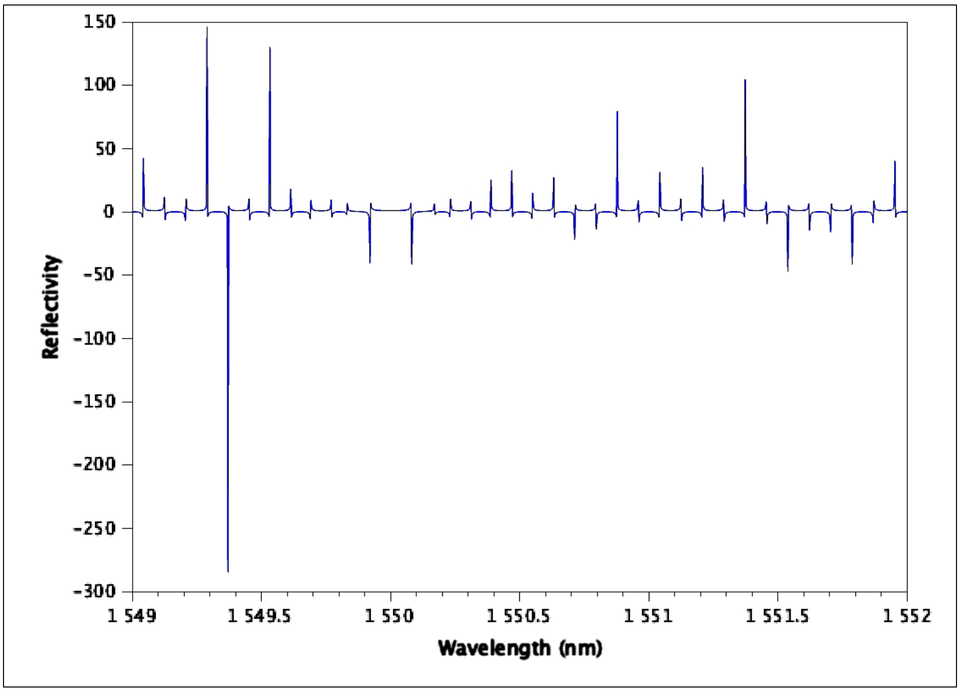


**Fig 1:** reflection spectra at different temperatures

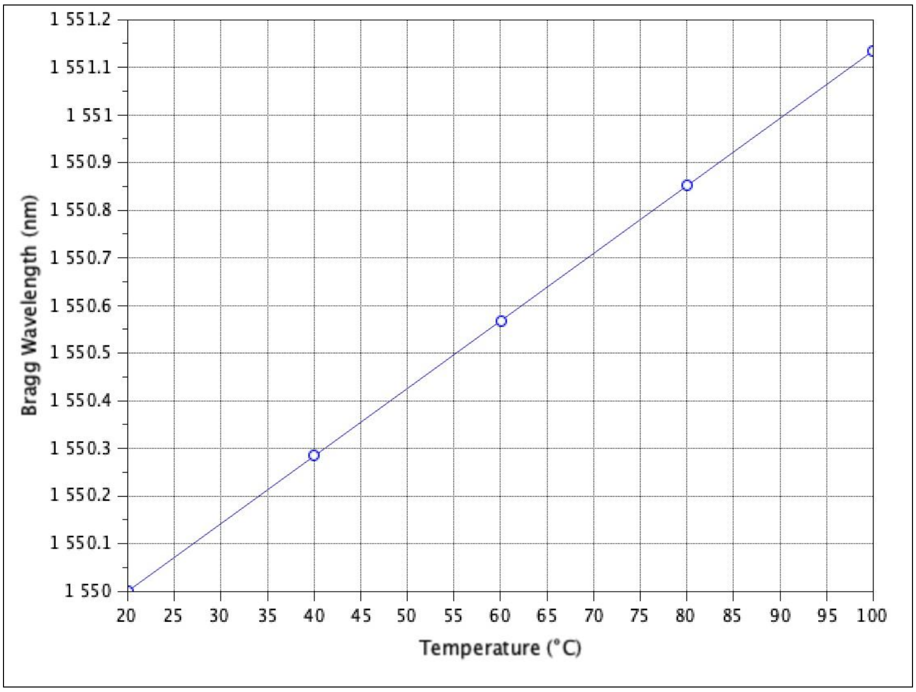


**Fig 2:** Bragg Wavelength Shoft vs Temperature

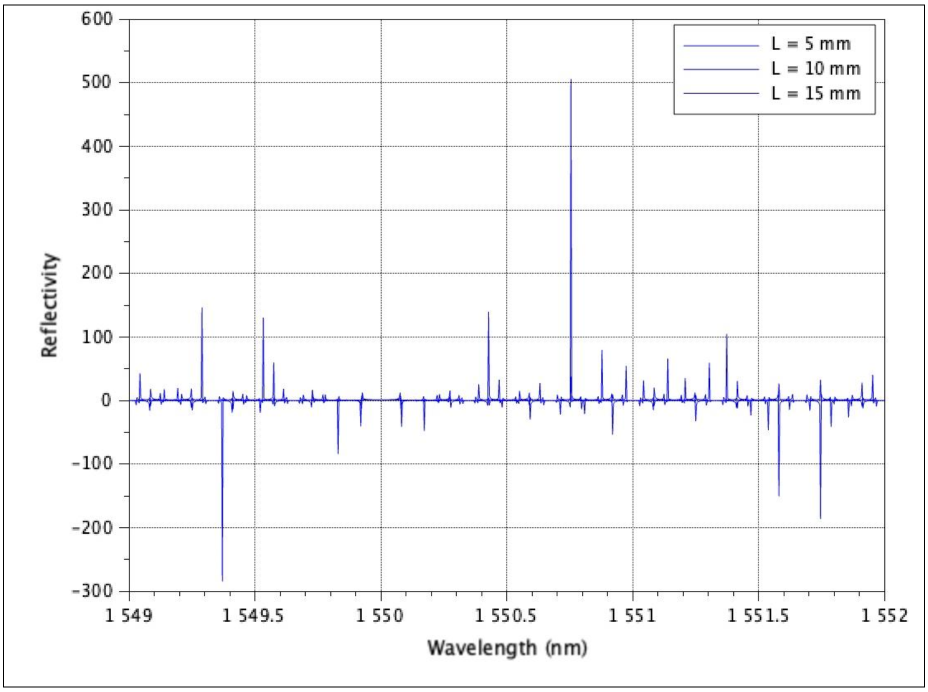


**Fig 3:** reflection spectra for different grafting lengths

## 5. Conclusion and Future Scope

The present study investigated the temperature sensing characteristics of Fiber Bragg Gratings (FBGs) through a theoretical framework based on Coupled Mode Theory (CMT) and validated the analysis using numerical simulations performed in Scilab. The results confirmed that the Bragg wavelength undergoes a linear red shift with increasing temperature, an effect arising from the combined influence of the thermo-optic coefficient and the thermal expansion coefficient of silica fibers. The simulated sensitivity of approximately 10.2 pm/°C matched closely with theoretical predictions and was consistent with experimental values reported in literature, thereby validating both the analytical model and the computational approach.

An important observation from the simulations was that while the length of the grating influenced reflectivity levels, it had no impact on the thermal sensitivity of the device. This finding reinforces the understanding that sensitivity is primarily governed by the intrinsic material properties of silica rather than geometrical parameters. Furthermore, the stability of the reflection spectra, which maintained their overall shape across the temperature range, demonstrated the robustness of FBGs as temperature sensors. Collectively, these results confirm that CMT provides a reliable analytical foundation for modeling temperature effects in Bragg gratings and that Scilab offers a cost-effective and versatile platform for photonic simulations, enabling sensor design and optimization with high accuracy.

Looking ahead, the scope for extending this work is considerable. Future studies could focus on apodized and chirped gratings, where the incorporation of Gaussian or raised-cosine profiles would suppress sidelobes, enhance selectivity, and broaden the range of sensing applications. Another promising direction involves multi parameter sensing, where temperature and strain effects are analyzed simultaneously, and thereby enabling cross-sensitivity characterization for applications in structural health monitoring. The inclusion of nonlinear effects such as Kerr-induced refractive index variations within the CMT framework could further expand its applicability to high-power photonic devices. Additionally, hybrid approaches that combine CMT with the Transfer Matrix Method (TMM) may offer greater accuracy for modeling complex grating geometries. Finally, experimental validation through the fabrication and testing of FBGs will be crucial to benchmark theoretical predictions and simulation outcomes against real-world performance.

By providing a clear theoretical model, a validated numerical methodology, and a practical outlook for future research, this study establishes a solid foundation not only for temperature sensing but also for the broader design of FBG-based devices across domains such as telecommunications, aerospace, biomedical instrumentation, and energy systems.

**References**

1. Othonos A, Kalli K. Fiber Bragg Gratings: Fundamentals and Applications in Telecommunications and Sensing. Boston: Artech House; 1999.
2. Lee B. Review of the present status of optical fiber sensors. Opt Fiber Technol. 2003;9(2):57-79.
3. Rao YJ. In-fibre Bragg grating sensors. Meas Sci Technol. 1997;8(4):355-375.
4. Kashyap R. Fiber Bragg Gratings. San Diego: Academic Press; 1999.
5. Kersey AD, Davis MA, Patrick HJ, et al. Fiber grating sensors. J Lightwave Technol. 1997;15(8):1442-1463.
6. Othonos A. Fiber Bragg gratings. Rev Sci Instrum. 1997;68(12):4309-4341.
7. Srivastava SK, Tripathi R, Dutta S. Temperature sensitivity of polymer coated fiber Bragg gratings. Opt Commun. 2009;282(24):4841-4844