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Dr. Ranjan Prasad
Assistant Professor, Department of Physics, S.N.S.R.K.S. College, Saharsa, B.N. Mandal University Madhepura, Bihar, India

Corresponding Author:
Dr. Ranjan Prasad
Assistant Professor, Department of Physics, S.N.S.R.K.S. College, Saharsa, B.N. Mandal University Madhepura, Bihar, India

Characteristic study of high precision black hole scattering

Ranjan Prasad

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Abstract
Black holes represent the most extreme manifestations of gravity in our universe. The interactions presented by Black holes provide the profound insights into the nature of spacetime. Among these phenomena the black hole scattering is of increasing relevance due to its connection with gravitational-wave astronomy in the post-Minkowskian expansions and high-precision theoretical physics. Here we have presented a comprehensive study of the characteristic features of black hole scattering in the high-precision regime. Recent advances have elevated the predictive power of theory to the fifth post-Minkowskian order, revealing unexpected mathematical structures. These discoveries not only improve waveform modeling for gravitational wave observatories but also uncover new connections between general relativity, quantum field theory, and algebraic geometry. The spin effect sand finite mass-ratio corrections have also been discussed in this study.

Keywords: Black holes, scattering, gravitational- waveform, spin effects

1. Introduction

The study of black holes has transformed from a theoretical curiosity into a cornerstone of modern astrophysics and gravitational physics. Since the first direct detection of gravitational waves in 2015, compact-object interactions have become a primary tool for testing general relativity (GR) under strong-field conditions. The most gravitational-wave detections to date involve inspiraling binary black holes. The scattering of black holes along hyperbolic orbits has emerged as a complementary probe. Unlike bound inspirals, scattering encounters emphasize the role of high velocities and large impact parameters.

High-precision black hole scattering is not only of astrophysical interest but also of fundamental importance. It forms a bridge between classical GR, effective field theory (EFT), and modern scattering amplitude methods. By studying the deflection of two black holes as they interact gravitationally, one can derive observables such as the scattering angle, radiated energy, and recoil momentum. These observables are crucial for calibrating effective-one-body models used in gravitational-wave astronomy and for cross-validating analytical approaches with numerical relativity (NR). Recent years have witnessed remarkable progress. Analytic methods have extended post-Minkowskian calculations allowing predictions for scattering observables with unprecedented accuracy. At these orders, mathematical structures far richer than initially expected appear, involving elliptic functions and even Calabi-Yau periods. Such structures hint at deep connections between gravity and areas of mathematics associated with string theory and quantum field theory [1, 2, 3].

At the same time, NR simulations of black hole scattering provide a “ground truth” for testing analytic methods. Comparisons between 5PM predictions, EOB resummations, and NR simulations demonstrate remarkable agreement, highlighting the robustness of the analytic-numerical synergy. This synergy is especially vital as gravitational-wave detectors continue to improve, demanding increasingly accurate theoretical templates. Here we have presented black hole scattering as a critical testbed for general relativity and quantum-inspired approaches to gravity.

2. Theoretical Framework of Black Hole Scattering

2.1 Black Hole Scattering in General Relativity

In classical general relativity, the interaction between two massive objects can be studied under two broad regimes. Bound orbits which lead to inspirals and eventual merger of the objects. The unbound orbits that lead to scattering of the particles. In the scattering regime, the two black holes approach each other with an initial relative velocity, are deflected by their mutual gravitational attraction, and then move apart to infinity. The key observable is the scattering angle, defined as the net angular deflection of one body relative to its incoming asymptotic trajectory. Other observables include the radiated gravitational-wave energy, the linear momentum carried away by radiation which determines the recoil of the incident object. In this case there may be the time delay relative to free propagation [4]. Mathematically, black hole scattering is studied in the two-body problem of general relativity, where the exact solution remains unknown. However, several perturbative and approximate methods have been developed to compute the scattering observables with increasing precision.

The post-Minkowskian expansion is a key tool for describing scattering. Unlike the post-Newtonian expansion, which assumes small velocities, the PM expansion is valid for arbitrary velocities. It expands in powers of Newton's constant, while treating the velocities relativistically. Recent work has extended calculations up to 5PM, allowing high-precision predictions that can be compared against numerical relativity. The 5PM results are especially significant because they incorporate higher-loop contributions when derived via scattering amplitudes and reveal rich mathematical structures [5, 6].

2.2 Scattering Amplitude Approach

A modern breakthrough has been the application of quantum scattering amplitude techniques to classical gravity. The idea is that the classical two-body problem can be extracted from the low-energy or long-distance limit of quantum scattering amplitudes of gravitons [7, 8, 9].

This approach uses tools like: Generalized unitarity to compute multi-loop amplitudes. Double copy relations, which link gauge theory amplitudes to gravitational amplitudes. The Classical limit prescriptions, to separate classical contributions relevant to black holes from quantum corrections. Through this framework, researchers have been able to compute scattering observables at 3PM, 4PM, and 5PM orders, with unprecedented precision [10].

2.3 Effective-One-Body Formalism

The effective-one-body framework provides a unifying language to connect scattering data, in spiral dynamics, and gravitational-wave templates. EOB maps the two-body problem onto an effective particle of reduced mass moving in a deformed Schwarzschild (or Kerr, if spins are included) spacetime. The mapping allows scattering observables (like) to be incorporated into EOB potentials, thereby improving models used in waveform generation. With high-PM inputs, EOB models become increasingly accurate and extendable to regimes relevant for gravitational-wave detectors [11, 12].

2.4 Numerical Relativity (NR)

The numerical relativity simulations provide the most direct way of studying black hole scattering. By solving the full Einstein field equations on supercomputers, NR can produce precise values for scattering angles, radiated energy, and waveforms. Recent NR work has simulated scattering at different impact parameters and mass ratios, offering critical benchmarks for PM, eikonal, and EOB approaches. Notably, NR has validated analytic predictions up to 5PM and highlighted the limitations of perturbative expansions in ultra-strong-field regimes.

3. Characteristic Observables of High-Precision Black Hole Scattering: In the context of black hole scattering, the primary observables are the scattering angle, the radiated gravitational-wave energy, and the recoil velocity. Each carries distinct physical information and contributes to waveform modeling for gravitational-wave detectors. The interaction between two bodies is extensively given by the gravitational Einstein-Hilbert term:

$$S = -m_1 c \int ds_1 - m_2 c \int ds_2 - \frac{c^3}{16\pi G} \int d^4x \sqrt{-g}\, R[g] \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Variation of this action gives rise to the Einstein and geodesic equations. To explain our notation, the proper time intervals $ds_i = \sqrt{g_{\mu\nu} \dot{x}_i^{\mu} \dot{x}_i^{\nu}}\, d\tau$ give rise to the followed trajectories $x_i^{\mu}(\tau)$ ($\mu = 0, 1, 2, 3$) of the ith black hole, parametrized by a time parameter τ (a dot symbolizes a τ derivative). The spacetime metric $g_{\mu\nu}(x)$ yields the curvature scalar R[g] and g =det ($g_{\mu\nu}$) μν. [13]

3.1 Scattering Angle

The scattering angle measures the deflection of one body's trajectory relative to its incoming asymptotic direction. Recent 5PM calculations show that the scattering angle includes contributions from elliptic integrals and more exotic structures, far beyond polylogarithms, reflecting the complexity of multi-loop Feynman integrals in the amplitude approach. These results have been matched with NR, showing agreement within a few percent across a wide range of impact parameters [14]. At high precision, PM and amplitude-based approaches provide analytic expressions for up to 5PM order, at least in the small-mass-ratio limit. These results reveal structures involving Calabi-Yau periods, signaling deep connections with algebraic geometry.

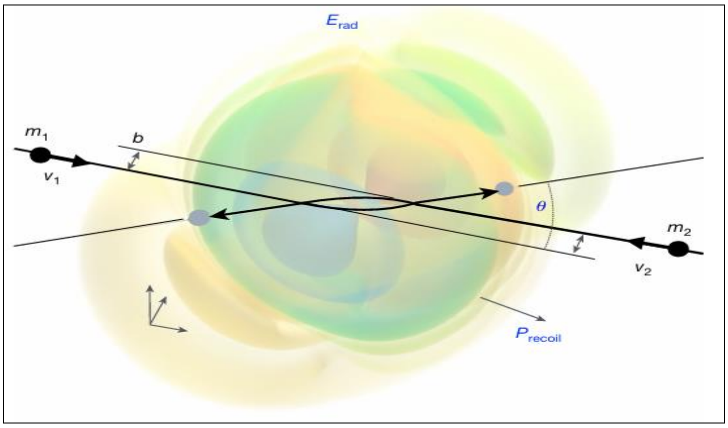

Fig 1: Two black holes scattering phenomenon.mi mis the mass of incoming black hole and vi is the incoming velocity, its impact parameter is b and the resulting relative scattering angle is θ, the radiated gravitational-wave energy is Erad.

Numerical relativity simulations confirm that the fraction of radiated energy depends strongly on the impact parameter. It provides a sensitive probe of the near-field dynamics and nonlinearities of GR.

In this study we have described a black hole scattering in bound state of two body problem. Here a weak-gravitational-field has been assumed, it is expanded in powers of Newton's constant G. It is valid as long as the two bodies are well separated and moving at arbitrary velocities [14, 15]. Here the fifth order (G5) precision has been employed to prepare for the third generation of gravitational-wave detectors[4].It is characterized by three fundamental properties of colliding particles such as their mass, spin and charge. Here the black holes are astrophysical equivalents of elementary particles. QFT is a highly mature subject and precise analytic predictions for particle scattering. This theory is used at CERN's Large Hadron Colliderand also at other places of collisions. This progress benefitted us in gravity through the close theoretical link between hyperbolic motion and elliptic motion. The technologies for performing the multi-loop Feynman integrals involved in scattering cross-sections have enabled us to predict some elementary particle physics [16].

A new landmark result has been reported here OF QFT-based classical general relativity programme by providing complete scattering observables of a binary black hole encounter up to the fifth order in the weak-field expansion (G_5) and sub-leading order in the symmetric mass ratio $\nu = m_1\ m_2/\ (m_1 + m_{2)})^2$. This encounter is presented in Fig. (1). It presents two black holes scattering with a deflection angle θ and radiating gravitational waves with total energy Read. We have presented the black holes as point particles. This approximation is valid as long as the separation b is large compared with their intrinsic sizes. Their Schwarzschild radii $2Gm_i/c^2$ valid in the weak-gravitational-field region. Therefore; the G expansion is really an expansion in the dimensionless quantity GM/bc^2 with total mass $M = m_1 + m_2$. The two scattering observables are θ and E_{rad}. Here Erad depends on CY3 periods. It can be used to calibrate gravitational-waveform models. Here, In Fig. (2) scattering angle θ is plotted as a function of the impact parameter in units of the Schwarzschild radius, bc^2/GM, up to order G^5 for an equal-mass scenario with initial relative velocity $v = 0.5125c$. In the figure the black dots are existing numerical relativity (NR) simulations[60]. The dashed line is the exact in G ($\nu = 0$) probe limit result for geodesic motion in a Schwarzschild background. The inset plot depicts the relative differences to the numerical relativity data. Larger values of bc^2/GM correspond to the perturbative regime. We find agreement with NR within the error for $bc^2/GM > 12.5$. The monotonically falling corrections to the consecutive G^n orders yield an intrinsic error estimate of our G^5 results: they are more precise than the NR data for $bc^2/GM > 14$.

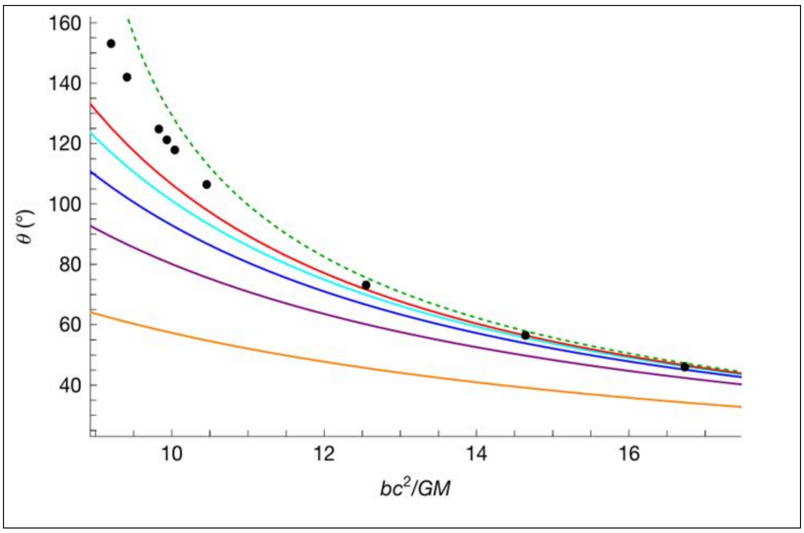

Fig 2: Graph of Scattering angle θ and impact parameter bc^2/GM

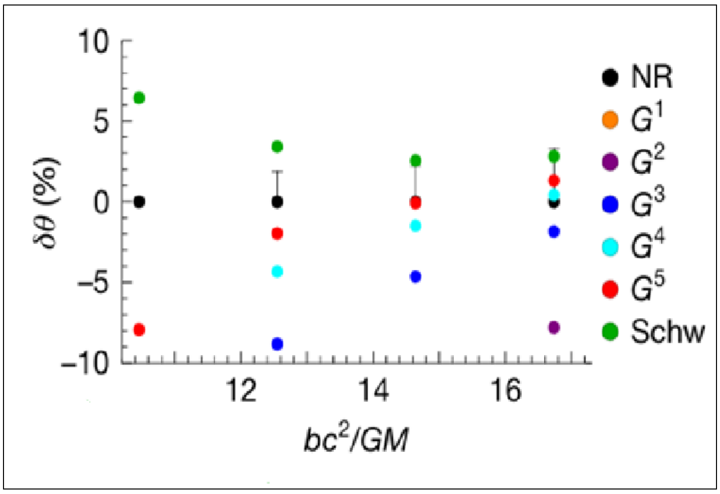

Fig 3: Graph of $\delta\theta(\%)$ and bc^2/GM

4. Methods of High-Precision Calculations: The study of black hole scattering relies on a blend of analytical techniques and numerical simulations. High-precision results arise from the cross-validation and synergy of these methods.

4.1 Post-Minkowskian (PM) Expansion
The PM expansion organizes calculations in powers of Newton's constant, rather than orbital velocity as in the post-Newtonian (PN) expansion. This makes PM particularly well-suited for unbound orbits like scattering.

4.2 Scattering Amplitude and EFT Approaches
Inspired by developments in quantum field theory (QFT), amplitude-based methods treat the two-body gravitational problem as a classical limit of scattering amplitudes.

4.3 Effective-One-Body (EOB) Formalism
The EOB approach recasts the two-body problem into that of a test particle moving in an effective spacetime. Originally developed for bound orbits, it has been extended to scattering.
The EOB framework is particularly valuable for gravitational-wave astronomy because it generates accurate waveform templates for detectors. For scattering, EOB models can interpolate between bound inspirals and hyperbolic encounters, ensuring continuity.

4.4 Numerical Relativity (NR)
NR solves Einstein's equations directly on supercomputers, without approximation. For scattering studies, NR plays three critical roles

1. **Validation:** Confirms the accuracy of analytic predictions (PM, EOB, amplitudes).

2. **Exploration:** Accesses highly relativistic regimes beyond perturbative reach.

3. **Calibration:** Provides data to fine-tune EOB models and resummations.

Recent NR simulations of hyperbolic black hole encounters have reached high precision, with accurate measurements of scattering angles, waveforms, and recoil velocities. These simulations confirm that PM and EOB results match remarkably well, even at moderate impact parameters.

5. Results and Comparative Analysis: High-precision studies of black hole scattering reveal a rich structure of observables, validated across multiple methods. Here, we summarize the key results for scattering angle, radiated energy, recoil, and mathematical structures, emphasizing the agreement and differences between PM expansions, amplitude methods, EOB formalism, and numerical relativity.

6. Comparison: PM predictions closely match NR simulations for moderate impact parameters.
EOB resummation improves agreement at small impact parameters by including non-perturbative effects. Discrepancies appear only in ultra-relativistic, strong-field scattering, where higher-PM terms and spins become significant.

Emergence of Mathematical Structures: One of the most striking discoveries in high-precision black hole scattering is the appearance of rich mathematical structures:

Elliptic Integrals (4PM): Arise in evaluating two-loop Feynman diagrams corresponding to classical interactions.

Calabi-Yau Periods (5PM): Unexpected structures in classical scattering observables, reflecting deep connections between gravity, string theory, and algebraic geometry.

7. Conclusion
High-precision black hole scattering has emerged as a pivotal area of gravitational physics, bridging classical general relativity, quantum-inspired techniques, and numerical relativity. This study has examined the characteristic observables scattering angle, radiated energy, recoil velocity, and time delay using a combination of post-Minkowskian (PM) expansions, scattering amplitude methods, eikonal approximations, effective-one-body (EOB) models, and numerical relativity (NR) simulations.

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