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# Limit cycle, gauge field and some unifications in particle physics

**Yi-Fang Chang**
Department of Physics, Yunnan University, Kunming, China

**Abstract**
First, various limit cycles in particle physics are searched. Next, we investigate the relations among the limit cycle and the oscillation-wave, and the limit cycles in the gauge field and the dynamical model, and the gauge field and the qualitative analysis theory, etc. Further, we research the limit cycle and some unified theories on particle. The limit cycle is also a special string. Some new mathematical results and equations are obtained. Finally, various possible developments of the limit cycle are proposed.

**Keywords:** particle physics; unification; limit cycles; interaction; gauge field; equation.

## 1. Introduction

In mathematics the limit cycle is defined by an isolated closed orbit, which may be stable, unstable or semi-stable. Therefore, we proposed that hadrons are similar to the limit cycles, whose exterior is an attractive strong interaction, and interior is a repulsive weak interaction which derives decay, both combination obtains hadron [1]. In particular, the stable limit cycle corresponds to stable proton. Further, hadrons are probably similar to the strange attractors [2]. Exterior of attractor all draws close it, which is the strong interaction with longer-range force; and interior of attractor all is repulsive each other, which is the weak interaction with shorter-range force. A zero dimensional strange attractor becomes a point charge, and corresponds to electron. Moreover, strange attractor possesses the many shell self-similar structures and fractal, which corresponds to the many shell-state model and the fractal model of particle [1].

Ordinary dynamical systems often have limit cycles, in which periodic orbits that are the asymptotic limit of generic solutions. From this Garfinkle explored the relationship of Choptuik scaling to the scale invariance of Einstein's equation, the periodicity of the scale-invariant part implies periodic self-similarity of the space-time [3]. Oliveira, *et al*., studied the general dynamics of the spherically symmetric gravitational collapse of a massless scalar field, which applied the Galerkin projection method, in which the critical solution between both asymptotic states represented by a limit cycle in the modal space [4]. Lechner, *et al*., analyzed a new transition between discrete and continuous self-similarity in critical gravitational collapse, in which two fixed points collide with a limit cycle in phase space [5]. Glazek used a new way to show renormalization group limit cycles of effective quantum theories [6]. Nishida formulated a field theory for resonantly interacting anyons, and derive the renormalization group equations, in which a limit cycle behavior in the four-body coupling implies an infinite set of bound states in the four-anyon system [7]. Glazek, *et al*., studied the relation between renormalization group and limit cycle [8].

Based on the universal wave-particle duality, along an opposite direction of the developed quantum mechanics, we applied a method where the wave quantities frequency v and wave length $\lambda$ are replaced on various mechanical equations, and derived some new results. It is called the mechanical wave theory. From this we derived new operators which represent more physical quantities. Further, we proposed some nonlinear equations and their solutions, which may be probably applied to quantum theory [9, 10]. We proposed mathematically the basic nonlinear operators and corresponding Klein-Gordon equation, Dirac equations, Heisenberg equation, etc. The present applied superposition principle is developed to the general nonlinear form. This theory may include the renormalization, which is the correction of Feynman rules of curved closed loops. We think the interaction equations must be nonlinear. Many theories, models and phenomena are all nonlinear, for instance, soliton, non-Abel gauge field, and the bag model, etc. The superluminal entangled state, which relates the nonlocal quantum teleportation and nonlinearity, should be a new fifth interaction. Moreover, the nonlinear effects exist possibly for various interactions, for single particle, for high energy, and for small space-time, etc. The relations among nonlinear theory and electroweak unified theory, and QCD, and CP nonconservation, etc., are expounded. We discussed some known and possible tests [1, 11]. We searched some new mathematical methods in particle physics, for example, quaternion, symbolic dynamics, theories on oscillation and particle with time, etc., and discussed the nonlinear theories and corresponding results of the qualitative analysis theory, and investigated the topological model and fractal model on particle, and proposed that the weak interaction corresponds possibly to Lobachevsky geometry [12]. In this paper, we search various limit cycles in particle physics, and obtain some new mathematical results and equations, and propose some possible developments of the limit cycle.

## 2. Oscillation-Wave and Limit Cycle

Characters of hadrons show that they are very analogy with the semi-stable or the double limit cycle [13]. When the systematic parameters change suitably, it resolves generally into two limit cycles: one stable and another unstable. The stable limit cycle at inner is analogy with the oscillation-rotation model (ORM), whose outer shell is the excite state, and core is particle at ground state [1]. Usual stable limit cycles are analogy with stable hadrons, in particular, nucleons. Both cycles correspond respectively to strong and weak interactions, from which some quantum numbers are conservation (stable) and unstable. Center is often a stable point (the equilibrium state). Probably, it corresponds to electron, and unstable parts correspond to meson-cloud. In particular, the transited section (phase transition point) from weak interaction to strong interaction is analogy with the stable limit cycle. The inside and outside regions of the limit cycle are topological separation. Its intersection points with axis are the fixed points.

Based on the gauge theory of various interactions in particle, we discussed some new solutions of the gauge field equations, and introduced the potential, and derived the relations among the results and the limit cycle, various singular points. We expounded possible physical meaning of property and phase transition of particle [14]. The gauge potential takes the anasatz condition:

$$A_a^\mu = \eta_{a\nu}^\mu \partial^\nu \varphi / \varphi = \eta_{a\nu}^\mu \partial^\nu \ln \varphi(x) \qquad (1)$$

The nonlinear equation of the gauge field with massless is:

$$\partial_\mu^2 \varphi + f^2 \varphi^3 = 0 \qquad (2)$$

The corresponding limit cycle is stable. If the gauge field has rest mass, equation will be [14]:

$$\partial_\mu^2 \varphi - m_0^2 \varphi + f^2 \varphi^3 = 0 \qquad (3)$$

In various figures the stable limit cycle may derive the stable focal point, which corresponds to point model (lepton, for example, electron e and neutrino v, etc.), or unstable focal point by Van der Pol method and double solutions [13]. The equations may be derived from group of interaction or model, and their solutions correspond to the equilibrium states and fixed points, and determine the limit cycles. Further, these may develop to attractor and strange attractor.

The linear damped oscillation corresponds to particles decay to electron, neutrino, and proton p. In these cases the singular points are the stable focal points, and correspond to e, v and p. For the ordinary differential wave equation

$$\psi'' + 2h\psi' + \omega^2 \psi = 0 \qquad (4)$$

Klein-Gordon (KG) equation may introduce analogously one order term. But, both are respectively continuous and quantized. Further, we should use that Dirac equations describe the decay process. If both combine, the one order term of Dirac equations may just describe decay.

Two differential equations with one order

$$Dx/dt = P(x,y), \quad dy/dt = Q(x,y); \qquad (5)$$

Represent the equilibrium state in phase plane x-y [13]. The intersection points ( $x_0, y_0$ ) between P(x,y)=0 and Q(x,y)=0 are namely the singular points of dy/dx=Q/P. Let

$$a = P_x'(x_0, y_0), \quad b = P_y'(x_0, y_0), \quad c = Q_x'(x_0, y_0), \quad d = Q_y'(x_0, y_0), \qquad (6)$$

For characteristic equation

$$\begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = 0, \qquad (7)$$

two roots $\lambda_1, \lambda_2$ have all negative real part, equilibrium states are stable; one root in $\lambda_1, \lambda_2$ has positive real part, equilibrium states are unstable; one root of real part is 0, then one approximate equation cannot solve the stable problem of equilibrium state. The isolated closed orbit forms the limit cycle [13]. In particle physics the stable central point, focal point correspond to neutrino and photon with m=0; while stable limit cycle corresponds to electron e and proton p, and may determine mass m or ratio of constant mc/h, etc.

At present only two-order or two dimensional ordinary differential equations have the limit cycle. Equations of interaction are one order partial differential equations, which may become two order ordinary differential equations. Collision is described by a set of equations or introduced potential, the section of the limit cycle corresponds to the collision cross-section. It is probably the structure of phase diagram.

Stable condition of particle $m^2/\lambda$<0 and $m^4/\lambda^2 > 16V/\lambda$ are replaced into $V=(\lambda/4)[\varphi^2-(m^2/2\lambda)]^2$, then

$$\varphi^2[\varphi^2-(m^2/\lambda)]<0, \tag{8}$$

Here $-(m^2/\lambda)$>0. If $\varphi^2 \geq 0$ is square of wave function, etc., so Eq. (8) is necessarily not hold. It may explain that all of meson and most particles are unstable. The existence of Eq. (8) must be $\varphi^2$<0 or $|m^2/\lambda|>|\varphi^2|$. For Dirac equations, etc., it may determine potential V.

The equation of harmonic oscillator is:

$$\ddot{x}+\omega_0^2 x=0. \tag{9}$$

Its solution is:

$$x=k\cos(\omega_0 t+\alpha),\ y=\dot{x}=-k\omega_0\sin(\omega_0 t+\alpha). \tag{10}$$

Such $x^2+(y^2/\omega_0^2)=k^2$ are a set of closed elliptical rings. The equation of repulsion is:

$$\ddot{x}+2h\dot{x}-nx=0, \tag{11}$$

Which has only the saddle point.

Duffing equation is [15]:

$$\ddot{x}+c\dot{x}+(\alpha x+\beta x^3)=F\cos\omega t. \tag{12}$$

F=0 corresponds to the nonlinear equation of quantum mechanics: c=0 corresponds to nonlinear KG equation and Higgs equation; $\ddot{x}$ =0 corresponds to nonlinear Dirac equation. The general equation is combined of nonlinear KG equation and Dirac equation for $\psi=\varphi$.

For C=special value, the solution of nonlinear KG equation is:

$$\varphi=(m_0/f)tg(m_0\eta/\sqrt{2}+C), \tag{13}$$

Which has periodicity. Suppose x=$\pi$/2, so $\eta=(\sqrt{2}/m_0)(\pi/2-C)$ is a phase transition point. Both sides are repulsion and gravitation, respectively, and both are topological separation. We may think that x<$\pi$/2 is a confined state. The solution of Higgs equation is a soliton of $\Gamma$-type (kink). For

$$\dot{\varphi}=(m_0^2/\sqrt{2}f)sc^2(m_0\eta/\sqrt{2}+C),\ \sqrt{2}\dot{\varphi}-\varphi^2=m_0^2/f^2. \tag{14}$$

At $\dot{\varphi}-\varphi$ plane it is parabola.

For $\varphi^2=y$, the solution of nonlinear KG equation is the soliton of $\Gamma$-type. The solution of Higgs equation is:

$$\varphi^2=2(m_0/f)^2[tg^2(m_0\eta+C)+1]. \tag{15}$$

It corresponds to the probability density. $\eta=(1/m_0)[(2k+1)(\pi/2)-C]$ Is the phase transition point. But, both sides all are repulsions. If sign is opposite, $m_0^2/f^2$<0, then it will be gravitation for $f^2\varphi^3$, and be the limit cycle. This will be more

complex.
The equations are [15]:

$$dx/dt = v\,;\ dv/dt = F(v) - x\,,\ F(v) = G(v) - av\,. \tag{16}$$

We can use $\varphi$ as characteristic curve, and obtain the limit cycle. $\varphi$ (y) →∞ is namely infinity of probability density. According to the uncertainty principle, the conjugate quantity of $\varphi$ (y) →0 , then radius, etc., will be determined. At present we discuss real space as x-y or corresponding momentum (P)-energy (E) plane. Probably, it corresponds to the extensive (super-) limit cycle, and is related with Lyapunov stability, etc. For the limit cycle, if force and potential, etc., are 0, their conjugate quantities will be infinity. Such we may investigate force, potential $\varphi$, field, quantum mechanics, and their equations and changes in x-y (or $\rho$ - $\varphi$ ) plane.

**3. Dynamical Model and Limit Cycle**

The ordinary differential equations of dynamical model (DM) [1] are:

$$\psi' + m\psi - a\varphi^2\psi = 0\,, \tag{17}$$

$$\varphi'' + 2a\overline{\psi}\psi\varphi - f^2\varphi^3 = 0\,. \tag{18}$$

Eq. (18) may resolve into a pair equations

$$\varphi' = y\,,\ \text{and}\ y' = f^2\varphi^3 - 2a\psi^2\varphi\,. \tag{19}$$

This may obtain three-dimensional, high-dimensional and strange attractor. By this method [13, 15], we introduce y' term and derive potential and equations.

$$(\dot{\varphi})^2 - (m\varphi)^2/4 + \lambda\varphi^4/4 = h\,, \tag{20}$$

Which is a stable ellipse. If $m^2 = 4\sqrt{\lambda}\dot{\varphi}$ , it will be a circle:

$$(\dot{\varphi} - \sqrt{\lambda}\varphi^2/2)^2 = (\dot{\varphi} - m^2\varphi/8\dot{\varphi})^2 = h\,. \tag{21}$$

Here h is square of circular radius, and corresponds to energy.
Harmonic oscillation and its model (DM may simplify to it) derive cycle. Outer damped motion of particle and inner anti-damped motion of particle all tend to cycle.
For Higgs breaking: when $A_\mu = 0$,

$$\gamma^\mu\partial_\mu\psi + m\psi - a\varphi^2\psi = 0. \tag{22}$$

From this derives $d\psi/d\eta' = a\varphi^2\psi - m\psi$ , here $\eta' = (\gamma_\alpha x_\alpha - u\gamma_4 t)/(1+u)$ .

$$\partial_\mu^2\varphi = -m_0^2\varphi + f^2\varphi^3\,. \tag{23}$$

Its integral is $p = d\varphi/d\eta = \pm\varphi\sqrt{f^2\varphi^2/2 - m_0^2}$ , here $\eta = (x - ut)/\sqrt{1-u^2}$ ; usually $\eta$ and $\eta'$ are not unity. But, if $\eta'$=c$\eta$ is unity, it will be one order equations:

$$\psi' = c(a\varphi^2 - m)\psi\,; \tag{24}$$

$$\varphi' = \pm\varphi\sqrt{f^2\varphi^2/2 - m_0^2}\,. \tag{25}$$

For dynamical breaking: when $A_\mu = 0$,

$$\gamma^\mu \partial_\mu \psi + m e^{b\varphi} \psi = 0. \tag{26}$$

$$\partial_\mu^2 \varphi = a e^{b\varphi}. \tag{27}$$

Both obtain:

$$(\partial_\mu^2 \varphi)(m\psi) + a\gamma^\mu \partial_\mu \psi = 0. \tag{28}$$

It becomes a pair equations with one-order:

$$\psi' = -cme^{b\varphi}\psi; \tag{29}$$

$$\varphi' = [2m\overline{\psi}\psi e^{b\varphi} + C]^{1/2}. \tag{30}$$

Let $e^{b\varphi} \approx 1 + b\varphi$, $\psi' = -cm\psi(1+b\varphi)$. For C=0, $\varphi' = \sqrt{2m\psi}(1+b\varphi/2)$. Both are approximation, it is existence at most of two limit cycles.

When $\varphi$ =0, the gauge theory is

$$\gamma^\mu(\partial_\mu + ig\gamma^5 A_\mu)\psi + m\psi = 0, \tag{31}$$

$$\partial_\nu F^{\mu\nu} = -\mu^2 A^\mu + ig\overline{\psi}\gamma^\mu\gamma^5\psi. \tag{32}$$

This includes quantum electrodynamics (QED) and quantum chromodynamics (QCD). For QED

$$\psi' = -m\psi - ig\gamma^\mu\gamma^5 A_\mu \psi, \tag{33}$$

$$\partial_\nu F^{\mu\nu} = -\partial_\mu^2 A^\mu,\ (A^\mu)' = [(\mu^2 A^\mu - 2ig\overline{\psi}\gamma^\mu\gamma^5\psi.)A^\mu]^{1/2}. \tag{34}$$

For QCD, there are also the structure function $f^{abc}$, etc. Above equations have not the coupling terms, they will be equations of self-interaction, for example, Eq. (23).

**4. Qualitative Analysis Theory and Gauge Field**

Electron e in electromagnetic interaction corresponds to stable focal point, and neutrino v in weak interaction corresponds possibly to unstable focal point, and strong interaction is the limit cycle. Points correspond to linearized equations [15]:

$$dx/dt = a_{11}x + a_{12}y,\ dy/dt = a_{21}x + a_{22}y. \tag{35}$$

Here $T = a_{11} + a_{22}$, $\Delta = a_{11}a_{22} - a_{12}a_{21}$. 1. When $\Delta$ >0 and $T^2 - 4\Delta \ge 0$, it has two real roots. In this case an equation is the equation of electron:

$$\psi' = m\psi + a_{12}\varphi. \tag{36}$$

Another equation is the W (Z) equation by integral one-time. 2. When $T^2 - 4\Delta$ <0, it has two conjugate complex roots. T≠0 is stable (for T<0) or unstable (for T>0) focal point. For T=0, so $\Delta$ >0 is a central point, and forms a closed orbit. But, there are above equations only $\psi$ and $\varphi$ interaction terms exist in Lagrangian. Interactions are preconditions. In mathematics singular point passes through bifurcation mechanics to form a simple asymptotic stable limit cycle, and the stable limit cycle may be formed from multiple limit cycle [16]. This corresponds to electron and neutrino pass through bifurcation mechanics to form smoothly proton p, $\mu, v_\mu, \pi$ and so on in physics, while stable particles are formed from decay of unstable particles.

The limit cycle is applied to baryon, and is extended to fermion and boson, for example, meson and photon. Here Goldstone particle is unstable, which may extend to quark and Higgs particle, etc., are unstable. Electronic orbit in atom should approximately be a limit cycle, and metric exists. It is also a possible unification on quantum mechanics and general relativity [17]. Further, we discuss various relations among the limit cycle and the gauge field, nonlinear Schrodinger equation, and coupling equations. Higgs equation [18, 19] are:

$$\phi''+\frac{2}{r}\phi'-\frac{2}{r^2}\phi+m^2\phi-\lambda\phi^3=0\,. \tag{37}$$

The limit cycles derived from nonlinear gauge Schrodinger equation [20] correspond to baryon octet add e, v and photon $\gamma$, together 11. Some stable, some semi-stable or unstable limit cycles correspond to boson octet add $\gamma$, or $\pi^{\pm}, K^{\pm}, K^0$ add $\gamma$. The equation described Jackiw-Pi model is the nonlinear gauge Schrodinger equation [21, 22]:

$$i\hbar\frac{\partial\psi}{\partial t}=[-\frac{\hbar^2}{m}D^2+eA^0-g\rho]\psi\,. \tag{38}$$

Here $D=\nabla-\frac{ie}{\hbar c}A$, $E^i\equiv-\frac{1}{c}\partial_t A^i-\partial_i A^0=\frac{e}{kc}\varepsilon^{ij}J^j$, $B\equiv\varepsilon^{ij}\partial_i A^j=-\frac{e}{k}\rho$, $\rho=\psi^*\psi$.

For E$\geq$0, a stable Anasatz condition $\psi(t,x)\equiv\exp(-iEt/\hbar)e^{-\Lambda}\phi(x)$ is introduced, so equation becomes:

$$(4\bar{\partial}\partial+k^2-\eta)\phi=0\,,\ k^2=mE/\hbar^2\,. \tag{39}$$

Here $\eta\equiv 8(\bar{\partial}\Lambda)(\partial\ln\varphi)+(me/\hbar^2)A^0-\beta\rho$, which satisfy $\partial\eta=8(\bar{\partial}\Lambda)(\partial^2\ln\varphi)$. For a continuous solution of energy, the Anasatz condition $\varphi(x)=c(kz)^\lambda u(r)$ introduced, the equations are:

$$[\frac{d^2}{dr^2}+(2\lambda+1)\frac{1}{r}\frac{d}{dr}+k^2-\eta(r)]u(r)=0\,, \tag{40}$$

$$\frac{d\eta}{dr}=2r\frac{d\Lambda}{dr}\frac{d}{dr}[\frac{1}{r}\frac{d}{dr}\ln u]\,, \tag{41}$$

$$\frac{1}{r}\frac{d}{dr}[r\frac{d\Lambda}{dr}]=\beta e^{-2\Lambda}k^2(kr)^{2\lambda}u^2\,. \tag{42}$$

Let $A_a^\nu=\varepsilon_a^\nu\varphi(\eta)$, and $\eta=\frac{x-ut}{\sqrt{1-u^2}}$, so

$$\partial_\mu^2 A_a^\nu=\varepsilon_a^\nu\frac{d^2}{d\eta^2}\varphi=m^2A_a^\nu=m^2d\varphi\,. \tag{43}$$

For equation

$$a\varphi''+b\varphi\varphi'+c\varphi^3=0\,, \tag{44}$$

Assume that $\varphi'=y$, $y'=-(b\varphi y+c\varphi^3)/a$, so it is just Lienard equation:

$$f(x) = -a\varphi,\ g(x) = -d\varphi - c\varphi^3. \tag{45}$$

It is small relation with time, which becomes $\varepsilon\psi'$. The energy function is defined:

$$E(\varphi,\varphi') = \frac{1}{2}\varphi'^2 - \int_0^{\varphi}(d\varphi + c\varphi^3)d\varphi = \frac{1}{2}\varphi'^2 - (\frac{1}{2}d\varphi^2 + \frac{1}{4}c\varphi^4) = L. \tag{46}$$

From this we may discuss various cases: d=0 or d$\neq$0; c=0 or c=constant or variable; d/c<0 or d/c>0; symmetry unification and breaking, etc. Equation (44) may become:

$$\psi' + \beta\psi + \gamma\psi^3 = 0. \tag{47}$$

For example, the phase transition for *a*=0 and $\beta$=0 corresponds to neutrino v; for $\beta$>0 there is semi-stable state, there has self-interaction, and symmetry of left-right corresponds to neutron n and proton p; for $\beta$<0 there is Higgs field.

**5. Limit Cycle and Unification of Particles**

It is known that the most stable particles possess better SU (3) symmetry, and it is very successful that SU (3) and its broken are applied to the classification of particles and to the mass spectrum. Usual SU (3) symmetry is explained by hadrons as quark model. Based on the symmetry and dynamic breaking or Higgs breaking, from dynamical nonlinear equations and more general quantum mechanics equations of emergence string we derived the GMO mass formula [1, 23]:

$$M = M_0 + AS + B[I(I+1) - \frac{S^2}{4})], \tag{48}$$

And it's modified accurate mass formula [1, 23]:

$$M = M_0 + AS + B[I(I+1) - \frac{S^2}{2}]. \tag{49}$$

From (48) and (49) we may predict mass of for heavy flavor hadrons, for example, $m(\Xi_{cc}) = 3715$ or 3673MeV [24, 23]. In 10 July 2017 LHC announced to observe the new doubly charmed baryon $\Xi_{cc}^{++} = ucc$, whose mass is 3621.40MeV, which agree accurately our prediction and error only is 1.4% [25].

Further, we discussed Eq. (49) applied to heavy flavor hadrons, and derive some predictions, m($\Omega_{cc}^{+}$)=3950.7 or 3908*MeV*, and $m(\Xi_{bb})$=10396.8 or 10348.9*MeV*, etc. It is a quantitative and testable theory. Based on the new data, we proposed various lifetime formulas of heavy flavor hadrons, which very agree with experiments. This is a new method on lifetime of hadrons described by quantum numbers, and can be unified for mass and lifetime. We discussed an approximate simplified supersymmetry theory based on the known symmetrical particles and their excited states [26].

We reviewed various unified theories of interactions in particle physics. Then, based on the simplest unified gauge group GL (6,C) of four-interactions proposed by me [1], a possible form of Lagrangian in this scheme is researched. Some relations among these results and other unified theories are discussed, and the equations of different interactions are obtained [27]. At certain degree the grand unification theory is namely unification of the limit cycles and singular attractor. Limit of strong and weak interactions is namely electromagnetic interaction of positive and negative charges, whose mass m=0 corresponds to zero dimension. For different interactions the equations of quantum mechanics are different. Electromagnetic interaction is Abel group U (1), and strong and weak interactions are non-Abel groups SU (2) and SU (3), and are Yang-Mills (YM) field. Further, the cycle may extend to high dimensions. The non-Abel group equations of interactions with SU (N) symmetry obtain the period solution, which corresponds to the limit cycle.

The limit cycle may be derived from equation, which is determined by group and interaction, or contrarily equation is obtained from some phenomenal limit cycles. The limit cycle may be mathematically $\varphi - \dot{\varphi}$ and $\psi - \varphi$ (spinor field $\psi$ by differential one order corresponds to two order scalar field $\varphi$); or x-v, x-y (polar coordinates r-$\varphi$ plane). The physical meaning of the limit cycle may be: 1. Stable, semi-stable particles. 2. Unification and transformation of interactions. 3. Magnitude of hadrons. 4. Quark confinement. 5. Vacuum phase transition of QCD [28], etc.

The stability of the limit cycle corresponds to the stability of particles. Symmetrical places correspond to particle-antiparticle, to

different regions, to quantization and different masses. The semi-stable cycle is possibly analogy with general particles. Period motion corresponds to isolated closed trajectory, i.e., the limit cycle. Figure of inside and outside cycle is analogue, but both directions are opposite.
Two order equation is:

$$\psi''+2h\psi'+\omega^2\psi=0. \quad (50)$$

Its solution is:

$$\psi=e^{-ht}(Ae^{i\omega t}+Be^{-i\omega t}). \quad (51)$$

Characteristic exponent of the limit cycle is exp (-ht). It is stable for h>0, and corresponds to gravitation; it is unstable for h<0, and corresponds to repulsion.
Mathematically, for two order equation the limit cycles ≥4 loops [29]; and correspond to stable particles: v, e, p, n (or $\gamma$ ). The dividing lines of $\Gamma_1,\Gamma_2,\Gamma_3$ correspond to strangeness numbers S=0,-1,-2. Three order equation≥11 loops; and correspond to hadrons octet, or its half is 6 loops, and correspond to photon $\gamma, \nu_e, \nu_\mu$ , e, p, n.
Equations (5) correspond to equations of quantum mechanics with interaction:

$$d\psi/d\eta=P(\psi,\varphi), \quad (52)$$

And $\varphi$ by integral one order

$$d\varphi/d\eta=Q(\psi,\varphi). \quad (53)$$

Variable t corresponds to $a(x-ut)$ and $b(P-uE)$. Equations with interactions all may become nonlinear.
From this we may phenomenally determine the cycle radius of interactions. Radius of strong interaction~$10^{-13}$cm. Assume that weak interactions transfer through $W^\pm$-Z, whose mass ratios and $\pi^\pm$-$\pi^0$ are $W^\pm$ (80.4)/ $\pi^0$ (0.135) =595.6, and Z (91.2)/ $\pi^\pm$ (0.140) =651.4. Average of two ratios is 623.5, so the weak interaction radius~$10^{-13}$/623.5=1.604$\times 10^{-16}$cm. Assume that the transition radius of two interactions $r_0$=5$\times 10^{-16}$ cm, it is radius of the limit cycle in space, and corresponds to $m_0$ =28GeV.
For strong and weak interactions the potential (field) is:

$$\varphi=(m-m_0)g^2e^{-mr}/r. \quad (54)$$

Radius r (and m) passes through $r_0$ ($m_0$) change, $\varphi$ (force) becomes from negative to positive (i.e., gravitation becomes repulsion). Further, we may research the relations among solutions of field equations, the limit cycle and $\varphi$ , force, etc.
Solved nonlinear density equations of fermion and boson are also results, in which Dirac equations become ordinary differential equations. Group determines transformation, whose fixed point is namely the limit cycle. In quantum mechanics equations are usually a trivial solution $\varphi$ =0. Solutions determine interactions, for SU (N) this is YM equation. Their various solutions, in particular, the elliptic function solutions may simplify to the trigonometric function solutions with periodicity, and corresponds to the cycle. Cervero, *et al*., obtained the elliptic solutions of classical YM theory based on the SU (2) YM field equation become to [30]

$$d^2f/dy^2+f^3-f=0. \quad (55)$$

Unification of the limit cycle should combine various known unify theories, for example, GUT, group, the gauge field and so on. Further, it combines string, special closed string theory, and the limit cycle is namely a special string, is an emergence string. From this derives the elliptic function solutions. O (4) shown that particles are the spherical symmetry, and agree with four dimensional rotation group. For free bosons KG equation, Proca equation are:

$$\partial_\mu^2A_\mu-m^2A_\mu=0. \quad (56)$$

It may be virtual mass m, which corresponds also to unifying subluminal and superluminal. Equation (54) becomes ordinary differential equation

$$f''-m^2 f = 0. \quad (57)$$

Its solution is:

$$f = Ae^{mx} + Be^{-mx}. \quad (58)$$

For A=0, x→∞, f=B; for B=0, x→-∞, f=A. Let $y = a(x-ut)$, x=0 or C, t→∞, so y→-∞. In Higgs breaking m corresponds to the cycle radius and space region.

The limit cycle has some characters. All cycle violates point models and space translation symmetry. Cycle and $\varphi$-V figure are respectively different projection and cross section. Inside and outside of the cycle are symmetry, but both directions are opposite; $\varphi$-V is left-right symmetry. The cycle has also two figures on $\dot{\varphi}-\varphi$ and *a-b*. From this we may understand strong and weak interactions, and understand Higgs mechanism and mass breaking. YM field equations with SU (N) derive potential and correspond to Higgs equation, such Higgs particle belongs to non-Abel group and short-rang interactions. The limit cycle and potential may suppose that both are equal, but are different cross section; both are similar, and may be extended each other. For example, the cycle must have a cross section, and potential must have the mass cycle $m/\sqrt{\lambda}$. The vacuum solution and constant solution of nonlinear equations correspond to focal point and cycle, while soliton solution, etc., corresponds to strong and weak interactions. Equation $f''+2f(f-1)(f-2)=0$ has O (4) symmetry, and f=0, 1, 2 are respectively origin, unstable and stable cycles. Potentials of strong-weak interactions should be a figure with three peaks; present exact theory of electromagnetic interaction is an inverse ratio curve, its approximation may become soliton. Reversely, it is namely a trap. Finite value corresponds to the renormalization mass. At present the minimum scaling r→0, and V→∞, corresponds to divergence. Trap corresponds qualitatively to the attractive electromagnetic interactions; middle extrusive trap corresponds qualitatively to weak and strong interactions for inside and outside cycle.

Present YM equation for Abel field has a Coulomb solution, which corresponds to electromagnetic interactions. For non-Abel field it has a like-Coulomb solution, which corresponds to the short range interactions. YM equation extends to Higgs equation and the limit cycle, which may combine various solutions and corresponding potentials, and discuss their physical meanings and various interactions. Inside and outside of cycle are superluminal and subluminal, which correspond to weak and strong interactions, and to $(m^2 - m_0^2)$ masses of $\pi$ and $W^{\pm}$ (Z), and to Higgs equation and KG equation, in which combined mass $(m^2 - m_0^2)$ or $m_0$ is namely Higgs mass. YM equation all may obtain:

$$\partial_\mu^2 \varphi = C\varphi^3. \quad (59)$$

Its solution $\varphi$=0 is stable or unstable focal point. Let the simplest $\varphi = a + bf$, so

$$f'' = C(a^3 + 3a^2 bf + 3ab^2 f^2 + b^3 f^3). \quad (60)$$

It is equation with the special coefficient. Equation derives the solution $\varphi$; Lagrangian L or Hamiltonian H derives potential V, and $\partial V/\partial x_\mu$ is force. For hadron, inside of cycle corresponds to weak interaction, $\varphi$=0 is unstable focal point, which corresponds to Goldstone particle with zero rest mass. When $\varphi$=0, potential is constant, and corresponds to not-divergent finite quantity.

For equation

$$F'' + (F/z)^3/4 = 0, \quad (61)$$

Assume that $F(z) = e^{ay} f(y)$, and y=lnz/b, so

$$dF/dz = (dF/dy)(dy/dz) = (e^{ay}/bz)(af + f'), \quad (62)$$

$$d^2F/dz^2 = -e^{3(a-b)y} f^3/4. \quad (63)$$

Let 2*a*=b, then

$$f''-a^2 f + a^2 f^3 = 0. \quad (64)$$

$\varphi = \pm\sqrt{a/b}$ Is the limit cycle, so for $|\varphi| < \sqrt{a/b}, > \sqrt{a/b}$, it is inside and outside of the cycle, both all are V ($\varphi$)>0.

$$\varphi = \sqrt{a/b}th(-\sqrt{a/2}y + C),\ \dot{\varphi} = (-a/\sqrt{2b})[ch(-\sqrt{a/2}y + C)]^{-2}. \quad (65)$$

So $\varphi^2 - \sqrt{2/b}\dot{\varphi} = a/b$, (66)

Is parabola. Forms of the limit cycle should be figures on y(x,t) and $\varphi$, $\dot{\varphi}$, V($\varphi$).
Present equation (5) of the limit cycle may extend to one order partial differential YM equations:

$$\partial_\mu F^a_{\mu\nu} + g\varepsilon^{abc} A^b_\mu F^c_{\mu\nu} = 0,\ \partial_\mu A^a_\nu - \partial_\nu A^a_\mu = F^a_{\mu\nu} - g\varepsilon^{abc} A^b_\mu A^c_\nu. \quad (67)$$

Or it becomes two order ordinary differential equations:

$$g'' = 2gh^2/r^2,\ h'' = h(h^2 - 1 + g^2)/r^2. \quad (68)$$

General equations are:

$$\partial_\mu\partial_\mu A^a_\nu - \partial_\mu\partial_\nu A^a_\mu + g\varepsilon^{abc}[\partial_\mu(A^b_\mu A^c_\nu) + A^b_\mu(\partial_\mu A^c_\nu - \partial_\nu A^c_\mu)] + g^2(\varepsilon^{abc} A^b_\mu)^2 A^a_\nu = 0. \quad (69)$$

If $\partial A_\mu / \partial x_\mu$ =0, so

$$\partial^2_\mu A^a_\nu + g\varepsilon^{abc} A^b_\mu(2\partial_\mu A^c_\nu - \partial_\nu A^c_\mu) + g^2(\varepsilon^{abc} A^b_\mu)^2 A^a_\nu = 0. \quad (70)$$

If $\mu = \nu$, *a*=*b*=*c*, and $A_\nu$ with mass m, so

$$\partial^2_\mu A_\nu + g\varepsilon A_\nu \partial_\nu A_\nu + g^2\varepsilon^2 A^3_\nu - m^2 A_\nu = 0. \quad (71)$$

By soliton way, etc., it becomes ordinary differential equation:

$$A_\nu'' + g\varepsilon A_\nu A_\nu' - m^2 A_\nu + g^2\varepsilon^2 A^3_\nu = 0. \quad (72)$$

Let $G = \int_0^{A_\nu}(g^2\varepsilon^2 x^3 - m^2 x)dx = g^2\varepsilon^2 A^4_\nu/4 - m^2 A^2_\nu/2$, $F = \int_0^{A_\nu} g\varepsilon x dx = g\varepsilon A^2_\nu/2$, then equivalent equations are:

$$dA_\nu/d\eta = y - g\varepsilon A^2_\nu/2,\ dy/d\eta = m^2 A_\nu - g^2\varepsilon^2 A^3_\nu. \quad (73)$$

So the limit cycle is determined only by F(x).
Equation of the limit cycle is:

$$x'' + f(x)x' + g(x) = 0, \quad (74)$$

Which may be composed from Dirac equations and their one order differential; or from KG equation and its one order integral; or it is the best way that obtained from coupling equations become an equation. This may be spread from QED, QCD, and DM, etc. Generally, the gauge theory is $\varphi$-$\psi$ interaction, or $\psi$-$A_\mu$ interaction; and $\varphi$ and $A_\mu$ all are two order equation, and both similarities and differences may be compared each other.

The nonlinear KG equation is:

$$\varphi''-m^2\varphi+b\varphi^3=0. \quad (75)$$

Its integral obtains $\varphi'=\pm\varphi\sqrt{m_0^2-b_0\varphi^2/2}$, so

$$f(\varphi)\varphi'\mp\varphi\sqrt{m_0^2-b_0\varphi^2/2}\,f(\varphi)=0. \quad (76)$$

Combining (73) and (74) obtain

$$\varphi''+f(\varphi)\varphi'-[m^2\pm f(\varphi)\sqrt{m_0^2-b_0\varphi^2/2}]\varphi+b\varphi^3=0. \quad (77)$$

Based on the Levinson-Smith (LS) theorem, in this equation let f ( $\varphi$ ) is even function and satisfy: condition 1.

$$b\varphi^3-[m^2\pm f(\varphi)\sqrt{m_0^2-b_0\varphi^2/2}]\varphi=g(\varphi), \quad (78)$$

Which odd function, and use $\varphi\neq 0$

$$b\varphi^4-[m^2\pm f(\varphi)\sqrt{m_0^2-b_0\varphi^2/2}]\varphi^2>0. \quad (79)$$

Condition 2. $\int_0^\varphi f(\varphi)d\varphi=F(\varphi)$ Is odd function, and use $\varphi_0>0$ for $0<\varphi<\varphi_0$, and F ( $\varphi_0$ )<0, while for $\varphi\geq\varphi_0$, and F( $\varphi$ )≥0 and is monotone increasing. For example,

$$f(\varphi)=3a\varphi^2-c,\ F(\varphi)=a\varphi^3-c\varphi. \quad (80)$$

Its intersection points with $\varphi$ axis are $\varphi=0$, $\pm\sqrt{c/a}$. In this case $\Delta=-3ac<0$, the poles are ( $\pm\sqrt{3ac}/3a$ , $\mp 2c\sqrt{3ac}/9a$ ). $\varphi_0=\sqrt{c/a}$ All satisfy (76). Condition 3. $\int_0^\infty f(\varphi)d\varphi=+\infty$, and $\int_0^\infty g(\varphi)d\varphi=+\infty$, so Equation has single limit cycle, and it is stable. Various constants in g ( $\varphi$ ) may be part as 0. Nonlinear Dirac equations are:

$$\gamma_\mu\partial_\mu\psi+m\psi-a\psi^3=0. \quad (81)$$

By soliton way they may become ordinary differential equation

$$\psi'+m\psi+a\psi^3=0. \quad (82)$$

Equation (82) by differential one time is:

$$\psi''+m\psi'+3a\psi^2\psi'=0, \quad (83)$$

Whose second term is replaced by Eq. (82), then obtain

$$\psi''+3a\psi^2\psi'-m^2\psi-am\psi^3=0. \quad (84)$$

In this case condition 2 satisfied must subtract (82).

$$\psi''+(3a\psi^2-1)\psi'-(m^2+m_0)\psi-(am+a_0)\psi^3=0. \quad (85)$$

Then in equation the suitable values agree with LS theorem.
Dirac equations become two order KG equation

$$\partial_{\mu}^{2}\psi - m^{2}\psi + J(\psi) = 0, \tag{86}$$

Which adds again primary equation multiplication of a factor.

By $(\gamma_{\mu}\partial_{\mu} + m + a\psi^2)\psi = 0$ left multiplicand $(\gamma_{\mu}\partial_{\mu} \pm m \pm a\psi^2)$ obtains the general equation:

$$[\partial_{\mu}^{2} + \gamma_{\mu}\partial_{\mu}(m + a\psi^2) \pm (m + a\psi^2)\gamma_{\mu}\partial_{\mu} \pm (m^2 + 2am\psi^2 + a^2\psi^4)]\psi = 0. \tag{87}$$

It becomes ordinary differential equation:

$$\psi'' + 2(a\psi^2 + m)\psi' + (a\psi^2 + m)^2\psi = 0, \tag{88}$$

Which obeys LS theorem. The nonlinear term $a\varphi + b\varphi^3 + c\varphi^5$ of nonlinear KG equation becomes to $(A + B\varphi^2)^2\varphi$, which is namely Eq. (88). The nonlinear Dirac equations have a single stable limit cycle, which corresponds to proton p. If there is not nonlinear interaction, it will change a strange point (electron). For m=0 it corresponds to neutrino, and for Maxwell equation it corresponds to photon.
Particle as wave packet may be related with the limit cycle as wave packet. The limit cycle must be a set of equations, which are usually one-order. It corresponds to $\varphi$-$\psi$ field with interaction, specially, after $\varphi$ integral one order. Such we may discuss the meaning of the $\varphi$-$\psi$ plane, and which transforms to the space-time coordinate.
The limit cycle is analogy with nonlinear Schrodinger ordinary differential equation, and KG-Dirac ordinary differential equation superposition. For independent space or mass shell they are ordinary differential equation, which may obtain equation of the limit cycle. Moreover, nonlinearity is also related with chaos and soliton, etc.

## 6. Discussion and Conclusion

Equation of the limit cycle cannot be free particle equation, but it is possible for interactions. For hadron it should be QCD, YM field or DM equations; for lepton it is already point particle. For hadron (baryon, meson) they are respectively Dirac equations and KG equation, Proca equation with interactions; mesons are also field quantum and equations of interactions. The limit cycle is related with general stability problem. Moreover, it must combine the uncertainty principle.
Further extension is various limit cycles on dividing space of strong-weak interactions, on time of interactions, and on energy; the limit cycles on repulsion and attraction among positive-zero-negative charges in electromagnetic interaction; the limit cycles on repulsion and attraction between particles, between nuclei and electrons, between atoms and molecules, etc.; the limit cycles on stable structures for all interactions. Moreover, the limit cycle may be analogue with fractional oscillator, whose equation is:

$$x(t) = x(0) + \dot{x}(0)t - \frac{\omega^2}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1}x(\tau)d\tau. \tag{89}$$

It is usual equation for $\alpha$ =2. When $\alpha$ <2, the total energy is not constant, and decreases along time with internal damping. It will become a focal point, which corresponds to particle decay.
If equation of the limit cycle is nonlinear, it should be chaos, and have possibly double solutions with soliton and chaos [31]. The nonlinear equations with two order system all may become Lienard equation. The limit cycle is a period motion, and has already extensive structure. It corresponds to development from motion equation to structure equation, i.e., (x,v) extends to (x,y) space structure, to (p, E) limit cycle of momentum and energy conversation. They are all extensive space and phase space, etc. It is contrary from the limit cycle construct equations, which must exist in two quantities with interactions.
Method on the extensive limit cycle may apply to various transformations of attraction and repulsion. It may apply to various regions on confinement and separation, and two parts of topological separation, for subluminal and superluminal [1], etc. Further, dark matter as the Galactic halo show already the region of similar limit cycle [32].

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